Module 1: Assignments

Course Code: 25MT105

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Course: Calculus and Ordinary Differential Equations
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Assignment 1

1. Calculate the points of inflection and the intervals for concavity and convexity for the following functions.
   - $f(x) = x^3 - 6x^2 + 9x + 1$.
   - $f(x) = x^4$.
   - $f(x) = x + \sin x$.
   - $f(x) = \frac{x}{x^2+1}$.
   - $f(x) = x^4 - 4x^3 + 6$.
   - $f(x) = e^x$.
   - $f(x) = ax^2 + \ln x$.
   - $f(x) = \sin x$ in $(0, 2\pi)$.
2. A machine learning model's loss function is $L(\theta) = \theta^4 - 8\theta^3 + 18\theta^2 + 5$, where θ represents a model parameter. Determine where $L(\theta)$ is convex (concave up). Convexity is crucial because gradient descent optimization algorithms are guaranteed to find the global minimum only in convex regions.

3. A pandemic spread model uses the logistic function $P(t) = \frac{10000}{1+99e^{-0.5t}}$ for the number of infected individuals at time t (days). Find the inflection point of this curve. This point is epidemiologically significant as it indicates when the infection rate transitions from accelerating to decelerating—the optimal time for public health interventions.

Assignment 2

1. (Rolle's Theorem) Verify Rolle's Theorem for the function $f(x) = x^3 - 6x^2 + 9x$ on the interval [0, 3]. Show that all the conditions of Rolle's Theorem are satisfied and find all values of $c \in (0, 3)$ such that $f'(c) = 0$.

2. (Mean Value Theorem – Inequality Form) Let $f(x) = \ln x$ for $x > 0$. Use the Mean Value Theorem to prove that for any $a, b > 0$, $|\ln a - \ln b| \leq \frac{1}{\min(a,b)}|a - b|$.

3. (First Mean Value Theorem for Integrals / Average Value) For the function $f(x) = 4x - x^2$ on the interval $[0, 4]$,

   - Compute the average value of the function.
   - Show that there exists a number $c \in [0, 4]$ such that $\int_0^4 (4x - x^2)\, dx = f(c) \cdot (4 - 0)$, and find the value(s) of c.

4. (Second Mean Value Theorem for Integrals – Application Type) Let $f(x) = e^x$ and $g(x) = 2 - x$ on the interval [0,2]. Using the Second Mean Value Theorem for Integrals, show that there exists a number $c \in [0, 2]$ such that

   $\int_0^2 (2 - x)e^x\, dx = e^c \int_0^2 (2 - x)\, dx.$

   Find the value of the integral $\int_0^2 (2 - x)\, dx$ and explain why such a c must exist.

Assignment 3

1. (Reduction Formulae) Derive the reduction formula for $I_n = \int \cos^n x\, dx$ and use it to evaluate $\int \cos^5 x\, dx$.

2. (Improper Integrals) Evaluate the improper integral $\int_2^\infty \frac{1}{x^3}\, dx$.

3. Determine whether the integral $\int_0^{\infty} xe^{-4x}\,dx$ converges. If it converges, find its value.
4. Determine the convergence of $\int_0^1 \frac{\ln x}{x^p}\,dx$ for different values of p.
5. Rectification (Arc Length): Find the arc length of the curve $y = \frac{1}{2}x^2$ from x = 0 to x = 2.
