



**VIGNAN'S Foundations for Science, Technology & Research**  
**SCHOOL OF APPLIED SCIENCE & HUMANITIES**  
**DEPARTMENT OF MATHEMATICS & STATISTICS**

Course : B. Tech. Credits : 04  
Year/ Sem. : I Year II Sem. Academic Year : 2025-2026  
Subject : Calculus & Ordinary Differential Equations Subject Code : 25MT105  
Branch : CSE (Section-7, 14) & ECE, VLSI (Section -21) Regulation : R25

**MODULE BANK**

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1. (10 Marks)
  - a. What is a double integral?
  - b. What are the limitations involved in the evaluation of double integrals?
  - c. Evaluate  $\iint_R x \, dx \, dy$ , where R is a triangle with vertices (0, 0), (5, 1) and (1, 1).
2. (10 Marks)
  - a. Describe the process of change of order of integration in a double integral and its purpose.
  - b. Write the limits of the integral to find the area bounded by the curves  $y = x^2$ ,  $x = y^2$ . (No need to evaluate.)
  - c. Find the area bounded by  $y = x - 2$  and  $y^2 = 2x + 4$ .
3. (10 Marks)
  - a. State the geometric meaning of a triple integral  $\iiint dv$ .
  - b. Set up and evaluate the volume of the tetrahedron bounded by  $x + y + z = 1$  and the coordinate planes.
  - c. Evaluate the volume of a solid bounded by the curves  $x^2 + y^2 = 4$  and  $z = 3$ .
4. (10 Marks)
  - a. Define a triple integral.
  - b. Evaluate the integral  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$ .
  - c. Find the limits of integration and evaluate the triple integral:  $\iiint (x + y + z) dV$ , where V is the region bounded by planes  $x = 0, y = 0, z = 0, 2x + 3y + 4z = 12$ .
5. (10 Marks)
  - a. Solve the following ODEs:
    - i.  $\frac{dy}{dx} = \frac{x}{y}$
    - ii.  $\frac{dy}{dx} = \sin x \cos^2 y$ .

- b. Solve  $\frac{dy}{dx} = e^{x+y}$  with initial condition  $y(0) = 0$  and state any domain restriction on the solution.
- c. Solve  $\frac{dy}{dx} = y(1 - y)$ . Identify the equilibrium solutions, solve for the general solution, express  $y$  explicitly in terms of  $x$  and interpret the solution as the logistic function, commenting on its behavior as  $x \rightarrow \infty$ .
- 6. (10 Marks)**
- a. Solve  $\frac{dy}{dx} = \frac{x+y}{x}$ . Verify it is homogeneous, and determine the solution in terms of  $x$  and  $y$ .
- b. Solve  $(x^2 + y^2) dx - 2xy dy = 0$ . Confirm homogeneity. Identify the family of curves represented by the solution.
- c. Solve  $\frac{dy}{dx} = \frac{(y^2 - x^2)}{2xy}$  with initial condition  $y(1) = 2$ . Verify that the particular solution satisfies the ODE.
- 7. (10 Marks)**
- a. A radioactive substance has a half-life of 8 days. (i) Using the decay model find the percentage remaining after 24 days. (ii) How many days until only 10% of the original amount remains?
- b. A bacterial culture starts with 300 bacteria. After 4 hours there are 1200 bacteria. (i) Set up the growth model and find the growth constant. (ii) Find the population after 10 hours. (iii) Determine when the population first exceeds 50,000.
- c. The number of infected individuals in an early outbreak follows  $\frac{dI}{dt} = kI$  with  $I(0) = 50$ . After 6 days,  $I = 200$ . (i) Solve the ODE and find  $k$ . (ii) Predict the number infected after 15 days. (iii) Compute the doubling time.
- 8. (10 Marks)**
- a. A metal rod at  $200^\circ\text{C}$  is placed in a room at  $25^\circ\text{C}$ . After 10 minutes it cools to  $120^\circ\text{C}$ . (i) Use Newton's Law of Cooling to find the cooling constant  $k$ . (ii) Find the temperature after 30 minutes. (iii) Determine when the temperature will reach  $40^\circ\text{C}$ .
- b. A bowl of soup is placed in a room at  $20^\circ\text{C}$ . After 5 minutes the temperature is  $80^\circ\text{C}$ , and after 10 minutes it is  $65^\circ\text{C}$ . From the above scenario, derive the model equation to find temperature at a given time.
- c. A dead body is found at  $30^\circ\text{C}$  in a room maintained at  $20^\circ\text{C}$ . One hour later, the body temperature is  $27^\circ\text{C}$ . Assuming normal body temperature at death was  $37^\circ\text{C}$ , using Newton's Law of Cooling, determine the time of death (i.e., the time  $t < 0$  at which  $T = 37^\circ\text{C}$  relative to when the body was found).
- 9. (10 Marks)**
- a. Define operator  $D = \frac{d}{dx}$ . How can the equation  $y'' + 3y' + 2y = 0$  be written in operator notation?
- b. Use the operator method (inverse operator) to find the particular integral of  $(D^2 + 4)y = x^2$ .

- c. Solve  $m \frac{d^2s}{dt^2} + k \frac{ds}{dt} + cs = 0$  given that  $k^2 < 4mc$ . Find the solution when  $k$  is so small that  $\frac{k^2}{mc}$  is negligible.

**10. (10 Marks)**

- What is the auxiliary equation of a differential equation? What are the three possible types of roots it can yield.
- Find a particular integral for the equation  $y'' - 3y' + 2y = e^{3x}$ .
- Analyze the situation where the forcing function  $f(x)$  in  $ay'' + by' + cy = f(x)$  is already a part of the complementary function. How must the form of  $y_p$  be modified, and what is the mathematical reason for this adjustment.

**11. (10 Marks)**

- State the general form of the complementary function in this case of repeated root.
- If  $\frac{d^2x}{dt^2} + 4x = 0$  and  $x = 3$ ,  $\frac{dx}{dt} = 8$  at  $t = 0$ , find  $x$  in terms of  $t$ . What is the maximum value of  $x$ ?
- Evaluate the limit of the solution  $y(t)$  as  $t \rightarrow \infty$ , for the equation  $y'' + 2y' + 5y = 0$ .

**12. (10 Marks)**

- For the equation  $y'' + ky = \sin(\omega x)$ , what is the standard "trial" form for the particular integral  $y_p$ ?
- Solve:  $y'' + 2y' + y = \cos(2x)\sin(3x)$ .
- Evaluate the solution of  $y'' + 4y = e^{-2x} + \sin(2x) + \cos(x)$ . Explain the physical phenomenon represented by the growth of this solution over time and identify the condition under which this occurs.

**13. (10 Marks)**

- Define the term **Complementary Function** and explain its role in the general solution of a non-homogeneous equation.
- Solve:  $y'' + 4y = e^x + x$
- Explain whether the order in which you find  $y_c$  and  $y_p$  matters. Can a specific choice of constants in  $y_c$  affect the validity of  $y_p$ ? Analyze the linear independence of the terms.