Vignan's Foundation for Science, Technology and Research

Method of Undetermined Coefficients

Trial Particular Integral (PI) — Complete Reference Table

25MT105 & Calculus and Ordinary Differential Equations(CODE) • Semester II / Year I

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How to use this table: Given $ay'' + by' + cy = R(x)$, first solve the **Characteristic Equation (CE)** $am^2 + bm + c = 0$ for roots m_1, m_2. Then find the form of $R(x)$ in the table, check the *Condition* column against those roots, and write the corresponding Trial PI. For **sums** of terms on the RHS, apply the **Superposition Principle**: solve each sub-problem separately and add the PIs.

RHS Type	Form of $R(x)$	Condition	Trial PI y_p
1. Exponential Functions			
Exponential	ke^{ax}	a is **not** a root of CE	Ae^{ax}
Exponential	ke^{ax}	a is a **simple** root of CE	$Ax\,e^{ax}$
Exponential	ke^{ax}	a is a **repeated** root of CE	Ax^2e^{ax}
2. Constants (Polynomial of degree 0)			
Constant	k (i.e. $ke^{0\cdot x}$)	0 is **not** a root of CE	A
Constant	k	0 **is** a root (no y term in ODE)	Ax
3. Polynomial Functions $P_n(x)$			
Polynomial	$P_n(x)$ (degree n)	0 is **not** a root of CE	$A_nx^n + \cdots + A_1x + A_0$
Polynomial	$P_n(x)$	0 is a root of multiplicity s	$x^s(A_nx^n + \cdots + A_0)$
4. Sine and Cosine			
Sin only	$k\sin(\beta x)$	$\pm\beta i$ are **not** roots of CE	$A\cos(\beta x) + B\sin(\beta x)$
Cos only	$k\cos(\beta x)$	$\pm\beta i$ are **not** roots of CE	$A\cos(\beta x) + B\sin(\beta x)$
Sin or Cos	$k\sin(\beta x)$ or $k\cos(\beta x)$	$\pm\beta i$ **are** roots of CE (resonance)	$x\left[A\cos(\beta x) + B\sin(\beta x)\right]$

RHS Type	Form of $R(x)$	Condition	Trial PI y_p
5. Product: $e^{ax} \times \sin(\beta x)$ **or** $e^{ax} \times \cos(\beta x)$			
Exp×Trig	$e^{ax}\sin(\beta x)$	$a \pm \beta i$ are **not** roots of CE	$e^{ax}\big[A\cos(\beta x) + B\sin(\beta x)\big]$
Exp×Trig	$e^{ax}\cos(\beta x)$	$a \pm \beta i$ are **not** roots of CE	$e^{ax}\big[A\cos(\beta x) + B\sin(\beta x)\big]$
Exp×Trig	$e^{ax}\sin(\beta x)$ or $e^{ax}\cos(\beta x)$	$a \pm \beta i$ **are** roots of CE	$xe^{ax}\big[A\cos(\beta x) + B\sin(\beta x)\big]$
6. Product: $e^{ax} \times P_n(x)$			
Exp×Poly	$e^{ax}P_n(x)$	a is **not** a root of CE	$e^{ax}(A_n x^n + \cdots + A_0)$
Exp×Poly	$e^{ax}P_n(x)$	a is a root of multiplicity s	$x^s e^{ax}(A_n x^n + \cdots + A_0)$
7. Sum of Terms — Superposition Principle			
Sum	$R_1(x) + R_2(x) + \cdots$	Apply rules to each term independently	$y_p = y_{p_1} + y_{p_2} + \cdots$
Example	$5e^{3x} + 3\cos(2x)$	Split into 2 sub-problems	$y_{p_1} + y_{p_2}$
8. Product: $\sin(\alpha x)\cdot\cos(\beta x)$ **— Use Trig Identity First**			
Sin×Cos	$\sin(\alpha x)\cos(\beta x)$	Expand: $\frac{1}{2}\big[\sin((\alpha+\beta)x) + \sin((\alpha-\beta)x)\big]$	Apply sin/cos rules to each term
Sin^2 or Cos^2	$\sin^2(\beta x)$ or $\cos^2(\beta x)$	Expand: $\frac{1}{2}\big(1 \mp \cos(2\beta x)\big)$	Apply constant + cos rules; superpose
★ Golden Rules — Always Check Before Writing Trial PI			
Rule 1	Any RHS type	Check if a or $\pm\beta i$ is a root of the CE	If match $\Rightarrow$ multiply trial by x (or x^2 for repeated root)
Rule 2	Sin or Cos	RHS has *only* $\sin(\beta x)$ or *only* $\cos(\beta x)$	Still write both: $A\cos(\beta x) + B\sin(\beta x)$
Rule 3	Sum of terms	$R(x) = R_1 + R_2 + \cdots$	$y_p = y_{p_1} + y_{p_2} + \cdots$ (superposition)

Abbreviations: CE $= am^2 + bm + c = 0$ (Characteristic Equation) • a = exponent in e^{ax} • β = frequency in $\sin(\beta x)/\cos(\beta x)$ • $P_n(x)$ = polynomial of degree n • s = multiplicity of root • $A, B, A_0, \ldots, A_n$ = undetermined coefficients to be found by substitution