Chapter 5: Second Order Ordinary Differential Equations

Tutorial

Dr. D Bhanu Prakash

Unit 5: Tutorial

Author: Dr. D Bhanu Prakash

Vignan's Foundation for Science, Technology and Research
Vadlamudi - 522 213, Guntur, Andhra Pradesh, India.

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Chapter 5: Second Order Ordinary Differential Equations
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Problem Sets

1. Introduction: Homogeneous and Non-Homogeneous Equations

Problem 1.1: Classify the following differential equations as homogeneous or non-homogeneous:

(a) $y'' + 3y' - 4y = 0$

(b) $y'' + 2y' + y = e^x$

(c) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \sin x$

Problem 1.2: Verify that $y_1 = e^{2x}$ and $y_2 = e^{-3x}$ are solutions of the homogeneous equation $y'' + y' - 6y = 0$.

Problem 1.3: Show that if y_1 and y_2 are solutions of the homogeneous equation $y'' + p(x)y' + q(x)y = 0$, then $c_1y_1 + c_2y_2$ is also a solution (principle of superposition).

Problem 1.4: Determine which of the following pairs of functions are linearly independent:

(a) e^x, e^{2x}

(b) $\sin x, 2\sin x$

(c) x, x^2

Problem 1.5: A cantilever beam under load satisfies the differential equation

$$EI\frac{d^4y}{dx^4} = w(x)$$

, where y is deflection, E is Young's modulus, I is moment of inertia, and $w(x)$ is the distributed load. For a uniformly loaded beam, $w(x) = w_0$ (constant). Classify this fourth-order equation as homogeneous or non-homogeneous. When $w_0 = 0$ (no load), engineers analyze the homogeneous equation to study natural beam shapes and buckling modes in structural analysis.

2. Complementary Functions (Homogeneous Solutions)

Problem 2.1: Find the complementary function (general solution of the homogeneous equation) for $y'' - 5y' + 6y = 0$.

Problem 2.2: Solve the homogeneous equation $y'' + 4y' + 4y = 0$.

Problem 2.3: Find the general solution of $y'' - 2y' + 5y = 0$ (complex roots case).

Problem 2.4: Solve $4y'' + 4y' + y = 0$ and express the solution in terms of real functions.

Problem 2.5: In a mechanical system, free undamped oscillation of a mass $m = 2$ kg on a spring with stiffness $k = 8$ N/m is governed by:

$$m\frac{d^2x}{dt^2} + kx = 0 \quad \Rightarrow \quad 2x'' + 8x = 0$$

Solve this homogeneous equation to find the complementary function $x_c(t)$. The solution reveals the natural frequency $\omega_n = \sqrt{k/m} = 2$ rad/s of the system. This natural frequency is critical for avoiding resonance in structural engineering and machine design—if external vibrations match this frequency, catastrophic oscillations can occur.

3. Complete Solutions and Initial Value Problems

Problem 3.1: Solve the initial value problem: $y'' + 4y' + 3y = 6e^{-x}$, $y(0) = 2$, $y'(0) = 1$.

Problem 3.2: Find the solution of $y'' - y = 2x$ with $y(0) = 0$ and $y'(0) = 0$.

Problem 3.3: Solve $y'' + 9y = 18x$ subject to $y(0) = 1$ and $y'(0) = 3$.

Problem 3.4: Determine the complete solution of $y'' - 6y' + 9y = e^{3x}$ with $y(0) = 0$ and $y'(0) = 2$.

Problem 3.5: A drone's altitude controller maintains vertical position using the differential equation:

$$\frac{d^2h}{dt^2} + 4\frac{dh}{dt} + 5h = 5h_d(t)$$

where $h(t)$ is actual altitude and $h_d(t) = 10$ meters is the desired altitude. The drone starts from ground level ($h(0) = 0$) with zero initial velocity ($h'(0) = 0$).

(a) Solve the complete IVP to find $h(t)$.

(b) Determine the steady-state altitude as $t \to \infty$.

(c) Find when the drone reaches 9 meters (90% of target altitude).

This analysis determines response time and overshoot—critical parameters for stable flight control in autonomous aerial vehicles.
