Credits: Avanti Sankalp Program

Unit 4: Integration

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12M07.0 Revision

- Differentiation → rate of change
- Differentiation of $x^n = nx^{n-1}$
- Differentiation of $kx^n = k(nx^{n-1})$

Q. Find $\frac{dy}{dx}$

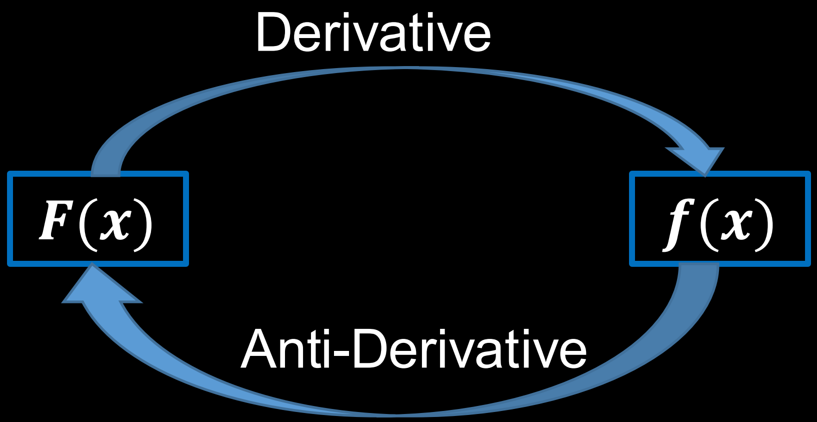
1. $y = 4x^3 + 7x + \frac{8}{x}$

2. $y = \sin^2 x - \cos x$

Sol.

1. $\frac{dy}{dx} = 12x^2 + 7 + 8\log x$

2. $\frac{dy}{dx} = \sin 2x + \sin x$



Integration

	A	B
Displacement (s)	$t^2 + 2t$	$t^3 + t^4$
Velocity(v)	$2t + 2$	$\boxed{3t^2} + \boxed{4t^3}$

Fill in the Blanks

$f(x)$	$F(x)$	$f(x)$	$F(x)$
$2x$		e^x	
$4x^3$		$\frac{1}{x}$	
$\cos x$		$2\cos x + x^2$	
$\sec^2 x$		$\sin 2x$	

Differentiation $\longrightarrow$ $\boxed{\dfrac{d}{dx}}$

Integration $\longrightarrow$ $\boxed{\int}$

$$\boxed{\frac{d}{dx}F(x) = f(x)}$$

Differentiation of $F(x)$ w.r.t x is equal to $f(x)$

$$\boxed{\int f(x)\,dx = F(x)}$$

Integration of $f(x)$ w.r.t x is equal to $F(x)$

Ex.

$$\frac{d}{dx}(\sin x) = \cos x \Rightarrow \int \cos x\,dx = \sin x$$

Integration of Standard Functions - Examples

Here are some examples of derivatives ,
try to find the anti-derivatives for same functions.

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(x^3) = 3x^2$$

$$\frac{d}{dx}\ln x = \frac{1}{\mathrm{x}}$$

$$\frac{d}{dx}e^x = e^x$$

$$\int \cos x \, dx = \sin x$$

$$\int 3x^2 \, dx = x^3$$

$$\int \frac{1}{x} dx = \ln x$$

$$\int e^x \, dx = e^x$$

Fill in the Blanks

$\frac{d}{dx}(x^2 + 5)$	? ?
$\frac{d}{dx}(x^2 - 100)$	? ?
$\frac{d}{dx}(x^2 - 1.6 \times 10^{-19})$	? ?

Fill in the Blanks

$\frac{d}{dx}(x^2+5)$ ⟵	$2x$
$\frac{d}{dx}(x^2-100)$ ⟵	$2x$
$\frac{d}{dx}(x^2-1.6\times 10^{-19})$ ⟵	$2x$

$$\frac{d}{dx}F(x) = f(x)$$

$$\int f(x)\,dx = F(x) + C$$

$$\int 2x\,dx = x^2 + C$$

Ex. Given $f(x) = 2x$, $\int f(x)dx = g(x)$, $\boxed{g(1) = 2}$, Find $g(x)$.

Sol.

$$\int f(x)dx = F(x) + C$$

$$\int 2x \; dx = x^2 + C$$

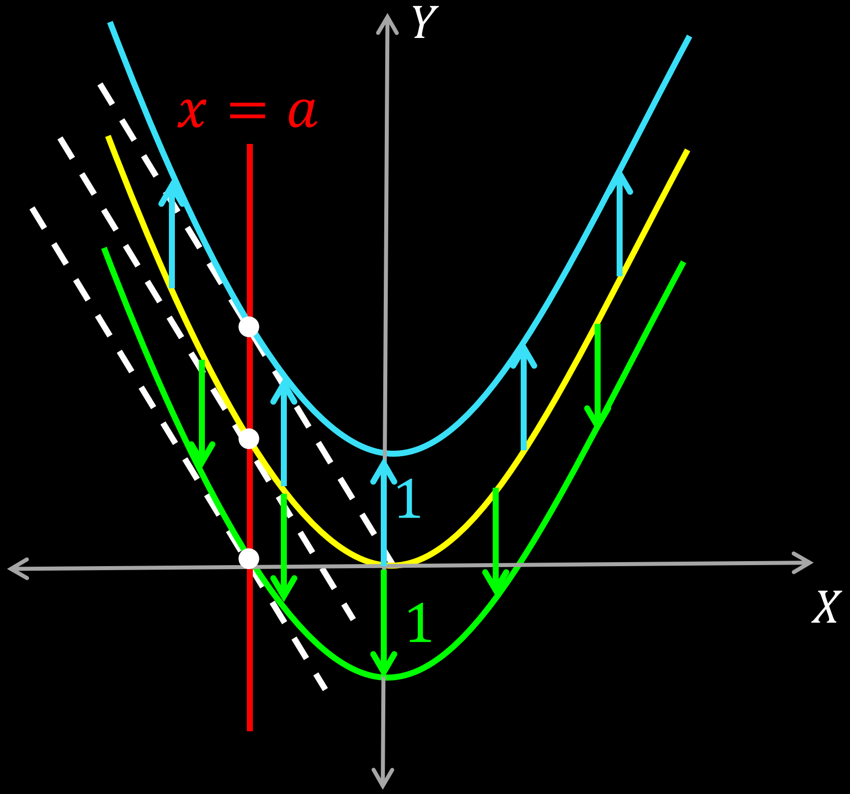
$$g(x) = x^2 + C$$

$$g(x) = x^2 + 1$$

$$g(1) = 1 + C = 2$$

$$C = 1$$

Geometrical Meaning of Constant of Integration



$y = x^2$

$y = x^2 - 1$

$y = x^2 + 1$

$$\frac{d}{dx}(x^2) = \frac{d}{dx}(x^2 - 1) = \frac{d}{dx}(x^2 + 1)$$

Fill in the Blanks

$f(x)$	$F(x)$
1	
x	
x^2	

$$\int x^n \, dx = \frac{x^{n+1}}{\boxed{n+1}} + C$$

$$n \neq -1$$

$$\int \frac{1}{x} \, dx = \log x + C$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\int \cos x = \sin x$$

$$\int \sin x = -\cos x$$

$$\int \sec^2 x = \tan x$$

$$\int \sec x = ??$$

$$\int \tan x = ??$$

Constant Rule:

$$\boxed{\int kf(x)dx = k\int f(x)dx}$$

Addition/Subtraction Rule:

$$\boxed{\int \big(f(x) \pm g(x)\big)dx = \int f(x)dx \pm \int g(x)dx}$$

Ex.

$$\int (2\cos x + x^2)\, dx = ?$$

$$2\int \cos x\, dx + \int x^2 dx = ?$$

$$(2\sin x + C_1) + \left(\frac{x^3}{3} + C_2\right)$$

$$2\sin x + \frac{x^3}{3} + C$$

$$\boxed{\int f(x)dx = F(x) + C}$$

$$\boxed{\int f(ax+b)dx = \frac{1}{a}F(ax+b)}$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \cos(2x+5)\, dx = \sin(2x+5) + C$$

$$\frac{d}{dx}\frac{(\sin(2x+5)+C)}{2} = \frac{2\cos(2x+5)}{2}$$

$$\int \cos(2x+5)\, dx = \frac{1}{2}\sin(2x+5) + C$$

Q. $$\int \frac{e^{2x} - 1}{e^{2x} + 1} dx$$

Q. $$\int \frac{e^{2x}-1}{e^{2x}+1}\,dx$$

Sol. $$\int \frac{e^{2x}-1}{e^{2x}+1}\,dx$$

Divide by e^{x}

$$\int \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}\,dx$$

Put, $t = e^{x}+e^{-x}$

$$dt = (e^{x}-e^{-x})dx$$

$$\int \frac{dt}{t} = \log|t| + C$$

$$= \log(e^{x}+e^{-x}) + C \quad ; \because e^{x} > 0$$

$$\log\left(e^{x}+\frac{1}{e^{x}}\right) + C$$

$$\log\left(\frac{e^{2x}+1}{e^{x}}\right) + C$$

$$\log(e^{2x}+1) - \log e^{x} + C$$

$$\log(e^{2x}+1) - x\log e + C$$

$$\log(e^{2x}+1) - x + C \quad , [\because \log e = 1]$$

$$\boxed{\frac{d}{dx}(e^{2x}) = 2e^{2x} \quad , \quad \frac{d}{dx}(1) = 0}$$

Q. $$\int \frac{x}{\sqrt{x+4}}\,dx$$

Q. $\int \frac{x}{\sqrt{x+4}}\,dx$

Sol. Put , $t^2 = x + 4$ or $x = t^2 - 4$

$$2t\,\frac{dt}{dx} = 1 + 0$$

$$2t\,dt = dx$$

$$\int \frac{t^2-4}{t}\,2t\,dt$$

$$2\int (t^2-4)\,dt$$

$$= 2\left[\frac{t^3}{3} - 4t\right] + C$$

$$= 2t\left[\frac{t^2}{3} - 4\right] + C$$

$$= 2\,t\left[\frac{t^2-12}{3}\right] + C$$

$$= 2\sqrt{x+4}\left[\frac{x+4-12}{3}\right] + C$$

$$= \frac{2}{3}\sqrt{x+4}\,(x-8) + C$$

Q1 $$\int \sin x \sin(\cos x)\, dx$$

Q2 $$\int (4x+2)\sqrt{x^2+x+1}\,dx$$

Q3 $$\int \frac{e^{\tan^{-1} x}}{1+x^2}\,dx$$

Q4 $$\int \frac{1}{\cos^2 x(1-\tan x)^2}\,dx$$

Q5 $$\int \frac{1}{1-\tan x}\,dx$$

Q1 $$\int \sin x \sin(\cos x)\, dx \quad = \cos\cos x + C$$

Q2 $$\int (4x+2)\sqrt{x^2+x+1}dx = \frac{4}{3}(x^2+x+1)^{\frac{3}{2}} + C$$

Q3 $$\int \frac{e^{\tan^{-1} x}}{1+x^2} dx \quad = e^{\tan^{-1} x} + C$$

Q4 $$\int \frac{1}{\cos^2 x(1-\tan x)^2} dx \quad = \frac{1}{1-\tan x} + C$$

Q5 $$\int \frac{1}{1-\tan x} dx \quad = \frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$$

Summary

- Let $P(x) = \int \frac{f'(x)}{f(x)} dx \;\; or \;\; Q(x) = \int f(x) f'(x)\, dx$

  Steps :-

  (I) Put $t = f(x)$

  (II) $dt = f'(x) dx$

  (III) Substitute $f'(x)dx$ with dt and $f(x)$ with t

  (IV) Integrate w.r.t t.

- Some standard Integrals :-

I. $\int tan\, x\, dx = log|sec\, x| + C$

II. $\int cot\, x\, dx = log|sin\, x| + C$

III. $\int sec\, x\, dx = log|sec\, x + tan\, x| + \mathrm{C}$

IV. $\int cosec\, x\, dx = log|cosec\, x - cot\, x| + C$

Revision

$2A$ Formulae

$$\sin 2A = 2\sin A\cos A$$

$$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

$$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$3A$ Formulae

$$\sin 3A = 3\sin A - 4\sin^3 A$$

$$\cos 3A = 4\cos^3 A - 3\cos A$$

$$\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$$

Product to Sum Formulae

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\cos B = \cos(A-B) + \cos(A+B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

Ex. $$\int \sin 6x \cos 3x \, dx$$

Sol.

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

Ex. $\int \boxed{\sin 6x \ \cos 3x} \, dx$

Sol. $\frac{1}{2}\int 2\sin 6x \cdot \cos 3x \, dx$

$[\because 2\sin A \cos B = \sin(A+B) + \sin(A-B)]$

$2\cos A \sin B = \cos(A-B) - \cos(A+B)$

$2\cos A \cos B = \sin(A+B) - \sin(A-B)$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$\frac{1}{2}\int (\sin(6x+3x) + \sin(6x-3x))dx$

$\frac{1}{2}\int (\sin 9x + \sin 3x)dx$

$= \frac{1}{2}\left[\frac{-\cos 9x}{9} - \frac{\cos 3x}{3}\right] + C$

$= \frac{-\cos 9x}{18} - \frac{\cos 3x}{6} + C$

Q1 $$\int \cos 2x \cos 4x \cos 6x \, dx$$

Q2 $$\int \frac{1 - \cos x}{1 + \cos x} \, dx$$

Q3 $$\int \sin^4 x \, dx$$

Q4 $$\int \frac{\cos x - \sin x}{1 + \sin 2x} \, dx$$

Q5 $$\int \frac{\cos 2x + 2 \sin^2 x}{\cos^2 x} \, dx$$

Q1 $$\int \cos 2x \cos 4x \cos 6x \, dx = \frac{1}{4}\left[\frac{\sin 12x}{12} + \frac{\sin 8x}{8} + \frac{\sin 4x}{4}\right] + C$$

Q2 $$\int \frac{1 - \cos x}{1 + \cos x} dx = 2\tan\frac{x}{2} - x + C$$

Q3 $$\int \sin^4 x \, dx = \frac{3x}{8} - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

Q4 $$\int \frac{\cos x - \sin x}{1 + \sin 2x} dx = -\frac{1}{\sin x + \cos x} + C$$

Q5 $$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \tan x + C$$

Summary

- Using Trigonometric Identities convert functions like :

$$\sin^2 x = \frac{1-\cos 2x}{2} \quad , \quad \cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin^3 x = \frac{3\sin x - \sin 3x}{4} \quad , \quad \cos^3 x = \frac{3\cos x + \cos 3x}{4}$$

- When Trigonometric ratios are given in product form, convert into sum form using these formulae

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A-B) + \cos(A+B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

$$\boxed{\sqrt{a^2 - x^2}}$$

put $x = a\sin\theta$ or $x = a\cos\theta$

$$\sqrt{a^2 + x^2}$$

put $x = a\tan\theta$ or $x = a\cot\theta$

$$\sqrt{x^2 - a^2}$$

put $x = a\sec\theta$ or $x = a\,\mathrm{cosec}\,\theta$

$$x = at$$

$$a^2 - a^2t^2$$

$$a^2(1 - t^2)$$

$$x = a\sin\theta$$

$$a^2 - a^2\sin^2\theta$$

$$a^2(1 - \sin^2\theta)$$

$$a^2\cos^2\theta$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}}$$

$$\boxed{\int \frac{dx}{\sqrt{x^2+a^2}}}$$

Put $x = a\tan\theta$; $\frac{x}{a} = \tan\theta$; $\tan\theta = \frac{\sqrt{x^2+a^2}}{a}$

$dx = a\sec^2\theta\, d\theta$

$$\int \frac{a\sec^2\theta\;\, d\theta}{\sqrt{a^2\tan^2\theta + a^2}}$$

$$\int \frac{a\sec^2\theta\;\, d\theta}{a\sec\theta}$$

$$\int \sec\theta\, d\theta$$

$$\log|\sec\theta + \tan\theta| + C$$

$$\log\left|\frac{x}{a} + \frac{\sqrt{x^2+a^2}}{a}\right| + C_1$$

$$\log\left|x + \sqrt{x^2+a^2}\right| - \log a + C_1$$

$$\log\left|x + \sqrt{x^2+a^2}\right| + C \quad ; \quad C = C_1 - \log a$$

Ex. $\displaystyle\int \frac{dx}{\sqrt{25 + x^2}}$

Sol. $\displaystyle\int \frac{dx}{\sqrt{x^2 + (5)^2}}$

Using Standard form $\displaystyle\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + C$

$$\log\left|x + \sqrt{x^2 + 25}\right| + C$$

Ex. $$\int \frac{dx}{1 - x^2}$$

Sol.

Ex. $$\int \frac{dx}{1 - x^2}$$

Sol. $$\int \frac{dx}{(1)^2 - x^2}$$

Put $x = a \sec \theta$

Using Standard form $$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a + x}{a - x} \right| + C$$

$$\frac{1}{2 \times 1} \log \left| \frac{1 + x}{1 - x} \right| + C$$

$$\frac{1}{2} \log \left| \frac{1 + x}{1 - x} \right| + C$$

Special forms of Integrals

$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left\|x + \sqrt{x^2 - a^2}\right\| + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left\|x + \sqrt{x^2 + a^2}\right\| + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

Special forms of Integrals

$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left\| \frac{x - a}{x + a} \right\| + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left\| \frac{a + x}{a - x} \right\| + C$
$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$	$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left\| x + \sqrt{x^2 - a^2} \right\| + C$
$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$	$\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left\| x + \sqrt{x^2 + a^2} \right\| + C$

Q. $$\int \frac{1}{\sqrt{7-6x-x^2}}\,dx$$

Sol.

Q. $$\int \frac{1}{\sqrt{7-6x-x^2}}\,dx$$

Sol. $$\int \frac{1}{\sqrt{(4)^2-(3+x)^2}}\,dx$$

Put, $t = 3 + x$

$$\frac{dt}{dx} = 0 + 1$$

$$dt = dx$$

$$\int \frac{dt}{\sqrt{(4)^2-t^2}} = \sin^{-1}\frac{t}{4} + C$$

$$= \sin^{-1}\frac{(3+x)}{4} + C$$

$$7-6x-x^2$$
$$= 7 - 2\times 3\times x - x^2$$
$$= 7 + 3^2 - 3^2 - 2\times 3\times x - x^2$$
$$= 16 - (3^2 + 2\times 3\times x + x^2)$$
$$= 16 - (3+x)^2$$
$$7-6x-x^2 = (4)^2 - (3+x)^2$$

$$\boxed{\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C}$$

Q1 $$\int \frac{1}{\sqrt{1+4x^2}}\,dx$$

Q2 $$\int \frac{3x}{1+2x^4}\,dx$$

Q3 $$\int \frac{x^2}{\sqrt{x^6+a^6}}\,dx$$

Q4 $$\int \frac{dx}{\sqrt{x^2+2x+2}}$$

Q5 $$\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$$

Q1 $$\int \frac{1}{\sqrt{1+4x^2}}dx = \frac{1}{2}\log\left|2x+\sqrt{4x^2+1}\right|+C$$

Q2 $$\int \frac{3x}{1+2x^4}dx = \frac{3}{2\sqrt{2}}\tan^{-1}(\sqrt{2}x^2)+C$$

Q3 $$\int \frac{x^2}{\sqrt{x^6+a^6}}dx = \frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right|+C$$

Q4 $$\int \frac{dx}{\sqrt{x^2+2x+2}} = \log\left|(x+1)+\sqrt{x^2+2x+2}\right|+C$$

Q5 $$\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = 5\sqrt{x^2+4x+10} -$$
$$7\log\left|(x+2)+\sqrt{x^2+4x+10}\right|+C$$

Summary

- When $(a^2 - x^2)$ or $\frac{1}{\sqrt{a^2-x^2}}$, then put $x = a\sin\theta$ or $x = a\cos\theta$
- When $(a^2 + x^2)$ or $\frac{1}{\sqrt{a^2+x^2}}$, then put $x = a\tan\theta$ or $x = a\cot\theta$
- When $(x^2 - a^2)$ or $\frac{1}{\sqrt{x^2-a^2}}$, then put $x = a\sec\theta$ or $x = a\,cosec\,\theta$

Summary

- $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left|\frac{x-a}{x+a}\right| + C$
- $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \log \left|\frac{a+x}{a-x}\right| + C$
- $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$
- $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + C$
- $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \frac{x}{a} + C$
- $\int \frac{1}{\sqrt{x^2+a^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + C$

Special forms of Integrals

$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a} + C$
$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left\lvert x + \sqrt{x^2 - a^2}\right\rvert + C$
$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \log\left\lvert x + \sqrt{x^2 + a^2}\right\rvert + C$

Principle

$\int \frac{1}{x} \log x \, dx$ Substitution

$\int \sin^3 x \, dx$ Trigonometrical Identities

$\int \frac{1}{(x+1)(x-2)} dx$ Partial Fractions

$$\int \sin^{-1} x \ln x \, dx = ?$$

$$\int x \, e^x dx = ?$$

ILATE rule

$\int U.V \; dx$

$$\int x \cos x \;\; dx$$

I	Inverse Function	$\sin^{-1} x$
L	Logarithm Function	$\ln x$
A	Algebraic Function	x^3
T	Trigonometric Function	$\sin x$
E	Exponential Function	e^x

$$\int x \sin^{-1} x \, dx$$

V U

I L A T E

Q. Find $\int x \cos x \, dx$

Sol.

Q. Find $\int x \cos x \, dx$

Sol.

$$\int \underset{U}{x} \, \underset{V}{\cos x} \, dx$$

$$x \int \cos x \, dx - \int \int cosx \;\, dx \left(\frac{dx}{dx}\right) dx$$

$$x \sin x - \int \sin x \, dx$$

$$x \sin x + \cos x + C$$

I L A T E

Q. Find $\int \log x \, dx$

Sol.

Q. Find $\int \log x \, dx$

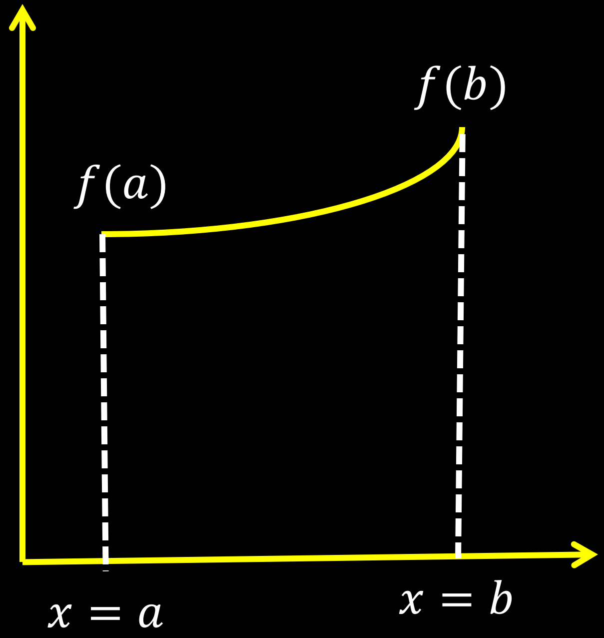
Sol.

$$\int \log x \, . \, 1 . \, dx$$

$$\log x \int 1 . \, dx - \int \frac{d(\log x)}{dx} \int 1 . \, dx \, dx$$

$$x \log x \; - \int \frac{1}{x} x \, dx$$

$$x \log x - x + C$$



$$F(x) = \int f(x)\,dx$$

$$F(x) = \int_a^b f(x)\,dx$$

Revision

If

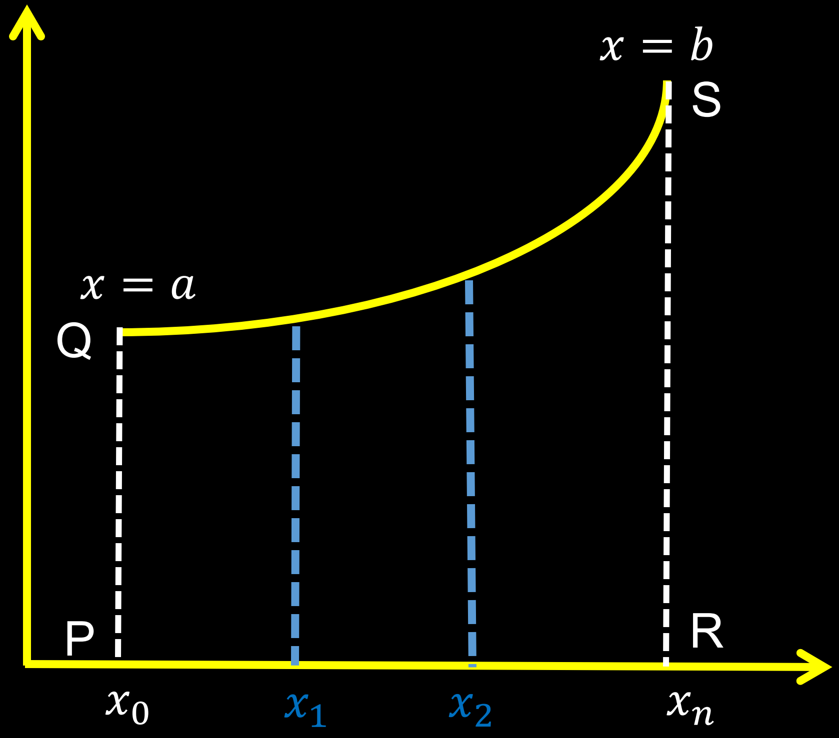
$$\lim_{n \to \infty} \frac{1}{n} = 0$$

$$n = \frac{1}{h} \qquad \lim_{h \to 0} h = 0$$

$$\lim_{n \to \infty} \frac{1}{n}$$

Principle

$$\int_a^b f(x)\,dx$$

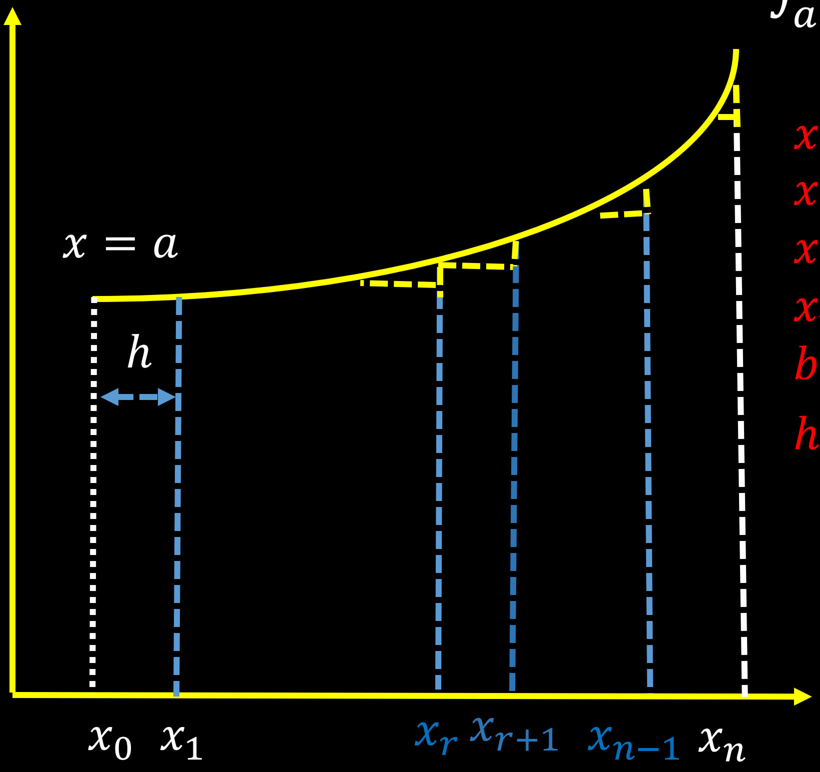


$$dA_0 = f(x_0) \times (x_1 - x_0)$$

$$dA_1 = f(x_1) \times (x_2 - x_1)$$

- Continuous Function
- Non – negative values, graph above the x-axis

The limit of the sum



$$\int_a^b f(x)dx$$

$x_0 = a$

$x_1 = a + h$

$x_2 = a + 2h$

$x_n = a + nh$

$b = a + nh$

$h = \frac{b-a}{n}$

$$dA_0 = f(x_0) \times h$$

$$dA_0 = f(x_0) \times (x_1 - x_0)$$

$$dA_1 = f(x_1) \times h$$

$$dA_0 + dA_1 + \cdots + dA_{n-1}$$

$$= h \times (f(x_0) + f(x_1) + \cdots + f(x_{n-1}))$$

$$dA_r = f(x_r) \times h$$

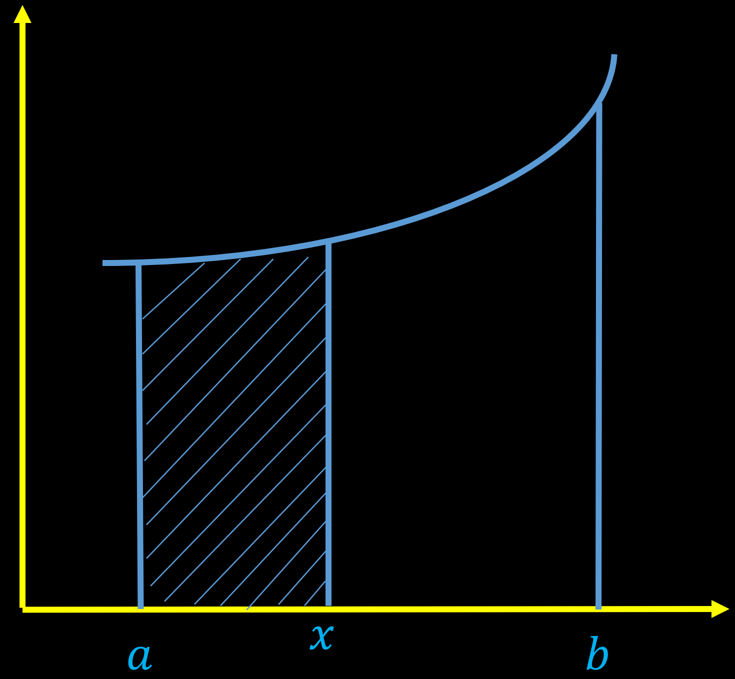
$$dA_{n-1} = f(x_{n-1}) \times h$$

$$\sum_{r=0}^{n-1} dA_r = h \sum_{r=0}^{n-1} f(x_r)$$

$$A = \lim_{n \to \infty} \left(h \sum_{r=0}^{n-1} f(x_r)\right)$$

$$\boxed{\int_a^b f(x)dx = \lim_{n\to\infty}\left(\frac{b-a}{n}\sum_{r=0}^{n-1} f(x_r)\right)}$$

Evaluation for Area Function



$\int_a^b f(x)\,dx$

$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$

$x \leftarrow [a, b]$

$\int_a^x f(x)\,dx$

- Continuous Function
- Well defined in $[a, b]$

Q. Find $\int_0^4 (x^2 + e^x)\, dx$

Sol.

$$\int_0^4 (x^2 + e^x)\, dx$$

$$\left[\frac{x^3}{3} + e^x\right]_0^4$$

$$\left[\frac{4^3}{3} + e^4\right] - [0 + e^0]$$

$$\frac{61}{3} + e^4$$

$$\int x^2\, dx + \int e^x\, dx$$

$$\frac{x^3}{3} + e^x$$

Q. Find $\int_0^1 3x^2(x^3+1)\,dx$

Sol.

$$\int_0^1 3x^2(x^3+1)\,dx$$

$$= \left[\frac{(x^3+1)^2}{2}\right]_0^1$$

$$= \frac{(1+1)^2}{2} - \frac{(0+1)^2}{2}$$

$$= \frac{4}{2} - \frac{1}{2}$$

$$= \frac{3}{2}$$

$$\int 3x^2(x^3+1)\,dx$$

$$x^3+1 = t,\ 3x^2dx = dt$$

$$\int t\,dt$$

$$\frac{t^2}{2} + \mathrm{C}$$

$$\frac{(x^3+1)^2}{2} + C$$