VIGNAN'S

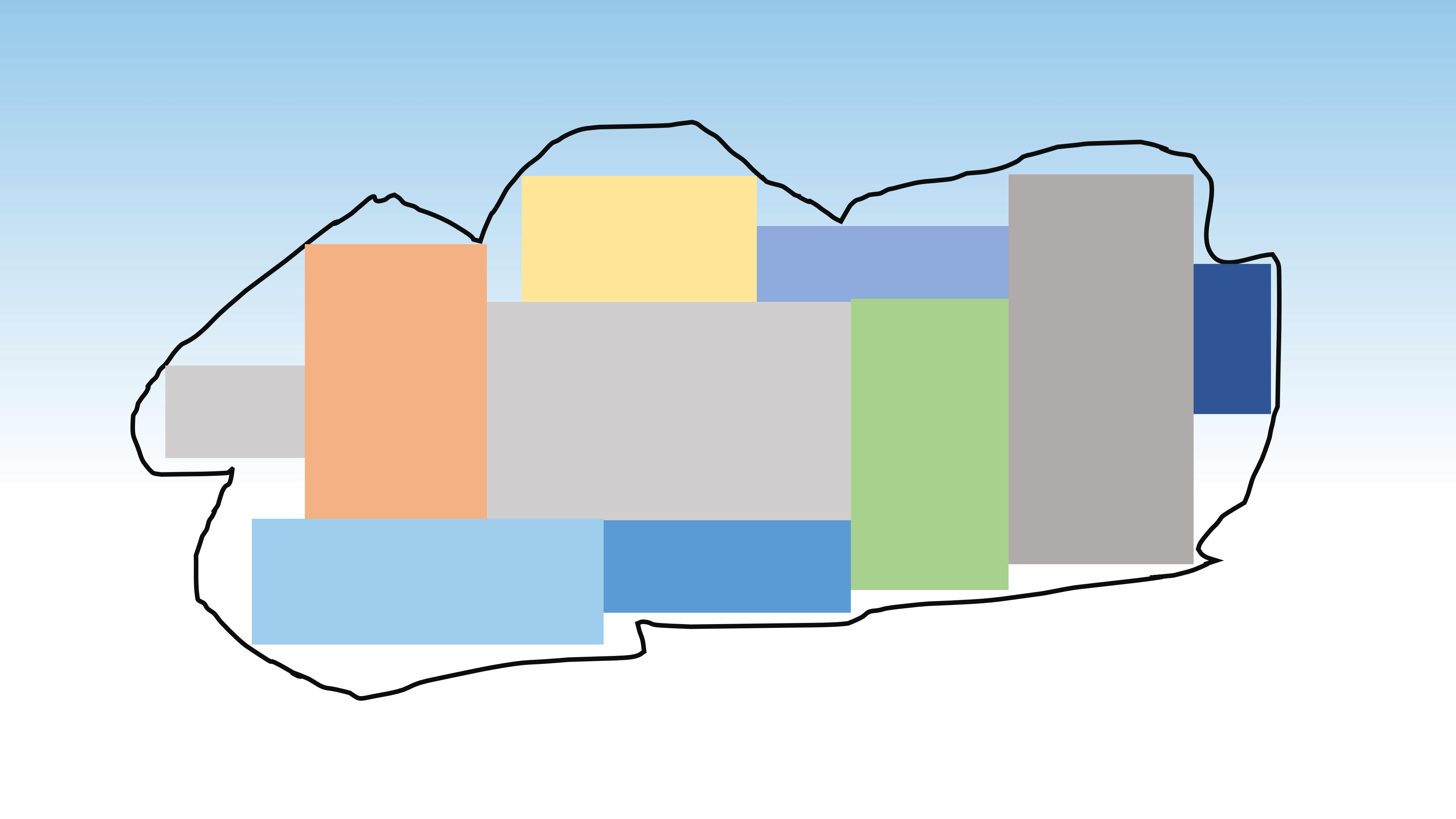
FOUNDATION FOR SCIENCE, TECHNOLOGY & RESEARCH

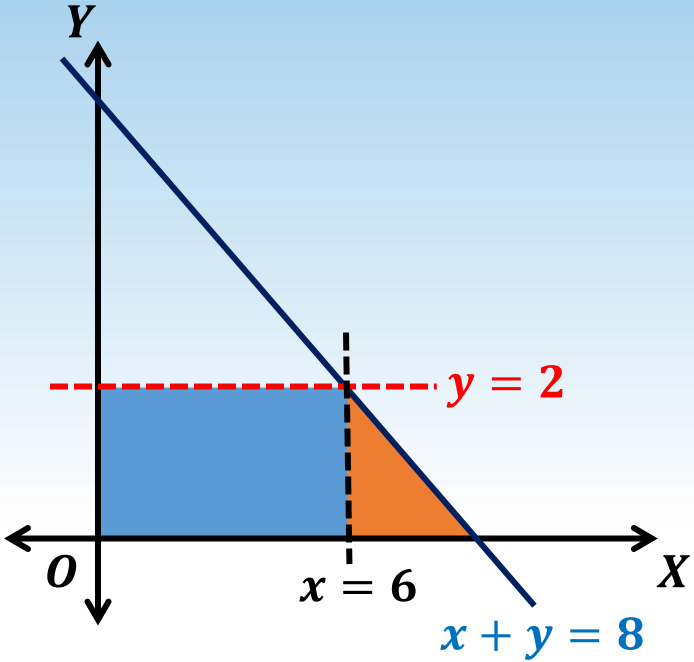
(Deemed to be University) - Estd. u/s 3 of UGC Act 1956

Credits: Avanti Sankalp Program

Unit 4: Integration - Applications

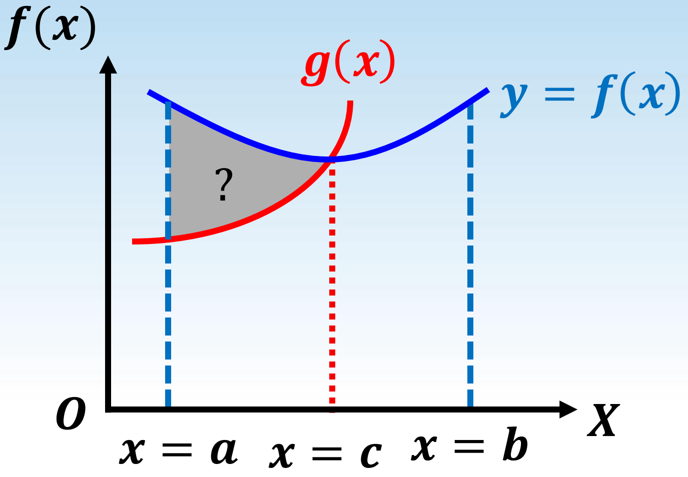
D Bhanu Prakash





Area $= 6 \times 2 + \dfrac{1}{2} \times 2 \times 2$

$=$ **14 Units**



Principle

Area $= \displaystyle\int_a^b f(x)\,dx$

Anti derivative of $f(x) = \mathbf{F}(x)$

$$\int f(x)\,dx = F(x) + C$$

$$\int_a^b f(x)\,dx = [F(x) + C]_a^b$$

Area Under a curve from $x = a$ to $x = b$ is $\int_a^b f(x)\,dx$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Q. Evaluate the **definite integrals,** if

1. $\int_0^1 \frac{dx}{1+x^2}$

2. $\int_0^1 xe^{x^2}\,dx$

Q. Evaluate the **definite integrals,** if

1. $\int_0^1 \frac{dx}{1+x^2}$

2. $\int_0^1 x e^{x^2}\, dx$

Sol.

$$= \int_0^1 \frac{1}{1+x^2}\, dx$$

$$= \left[\tan^{-1} x\right]_0^1$$

$$= \tan^{-1} 1 - \tan^{-1} 0$$

$$= \frac{\pi}{4} - 0$$

$$= \frac{\pi}{4}$$

$$= \int_0^1 e^{x^2} x\, dx$$

$$e^{x^2} = t$$

$$e^{x^2} 2x\, dx = dt$$

$$e^{x^2} x\, dx = \frac{dt}{2}$$

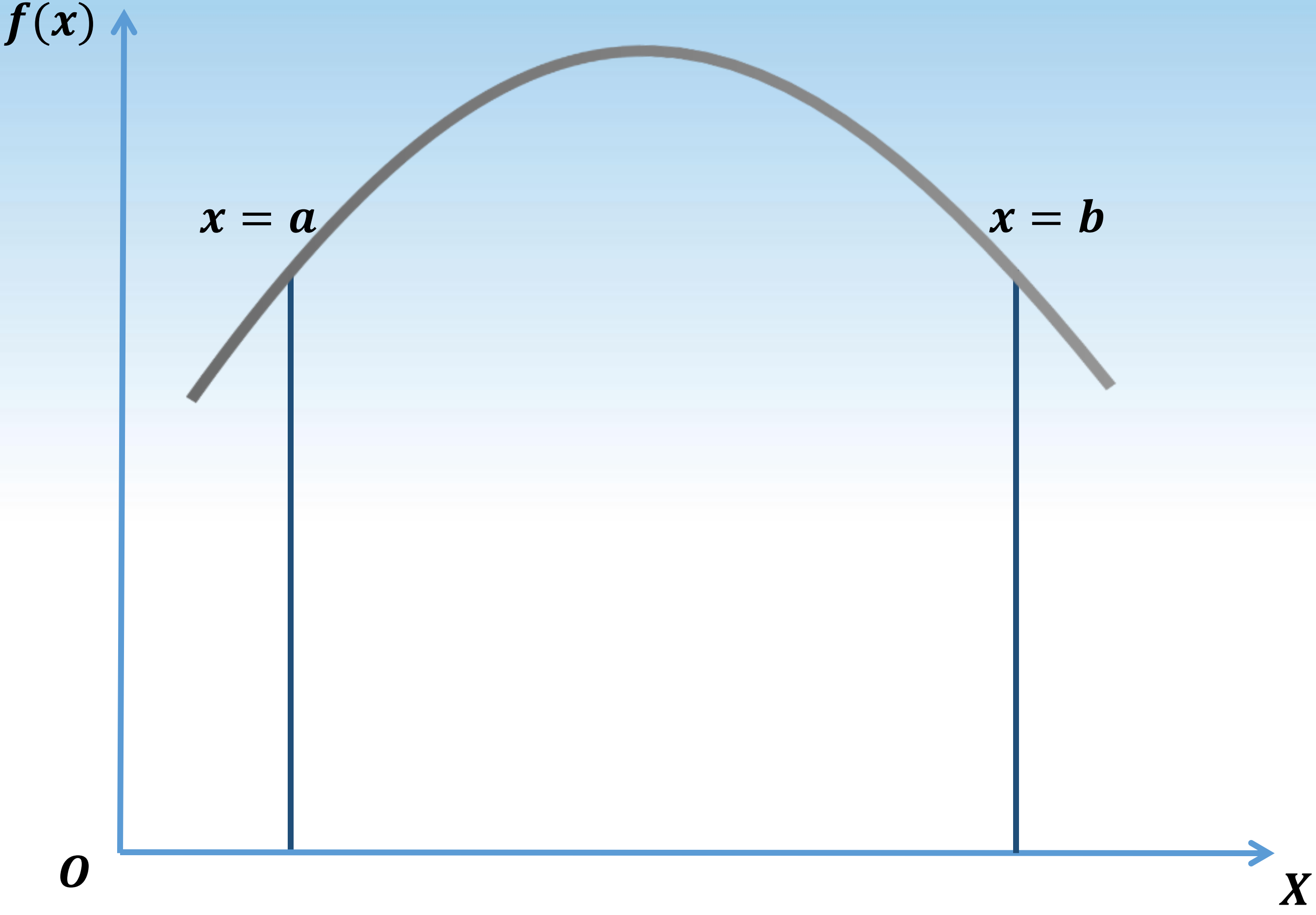
$x = 0 \quad t = 1$

$x = 1 \quad t = e$

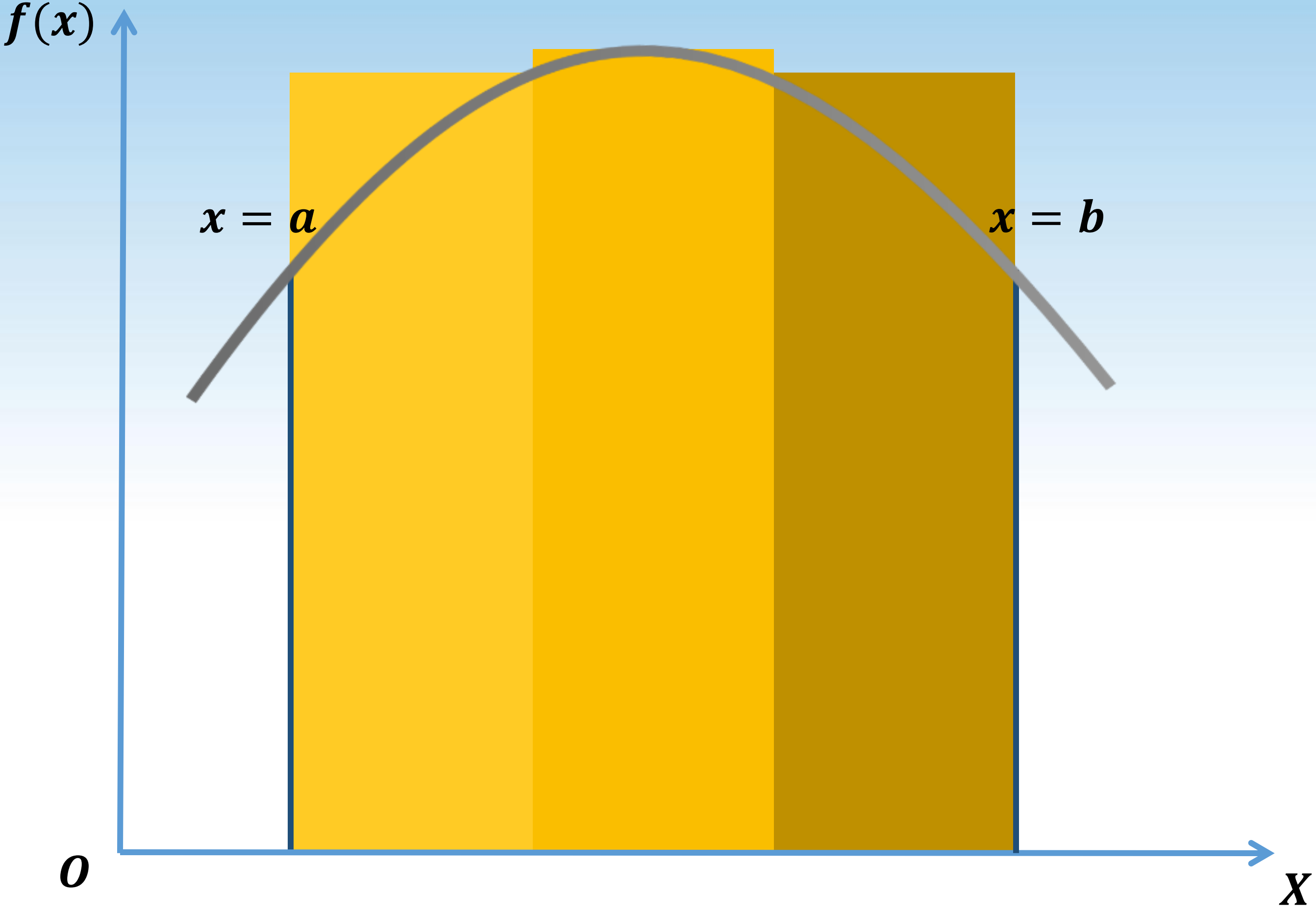
$$= \int_1^e \frac{dt}{2}$$

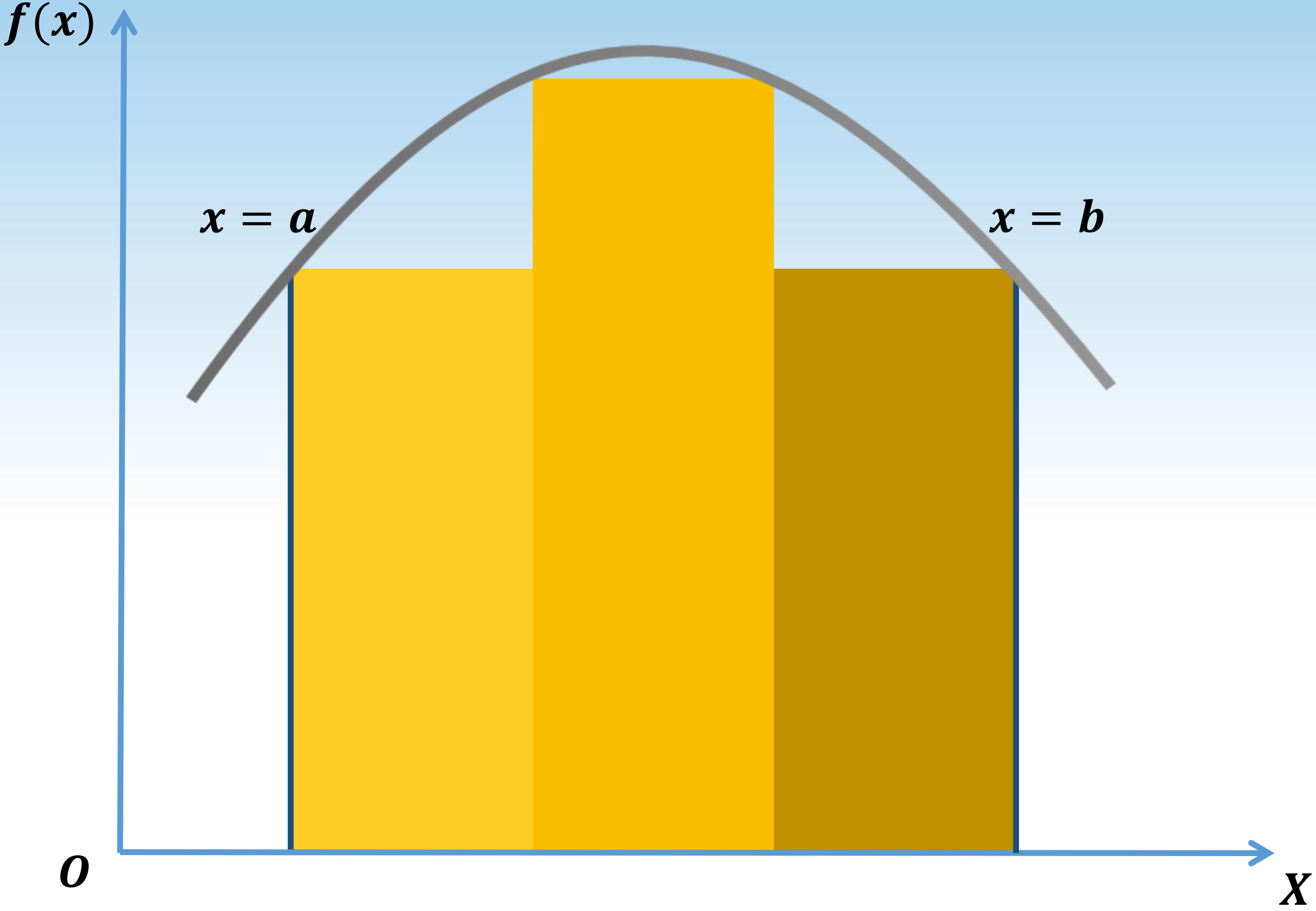
$$= \left[\frac{t}{2}\right]_1^e$$

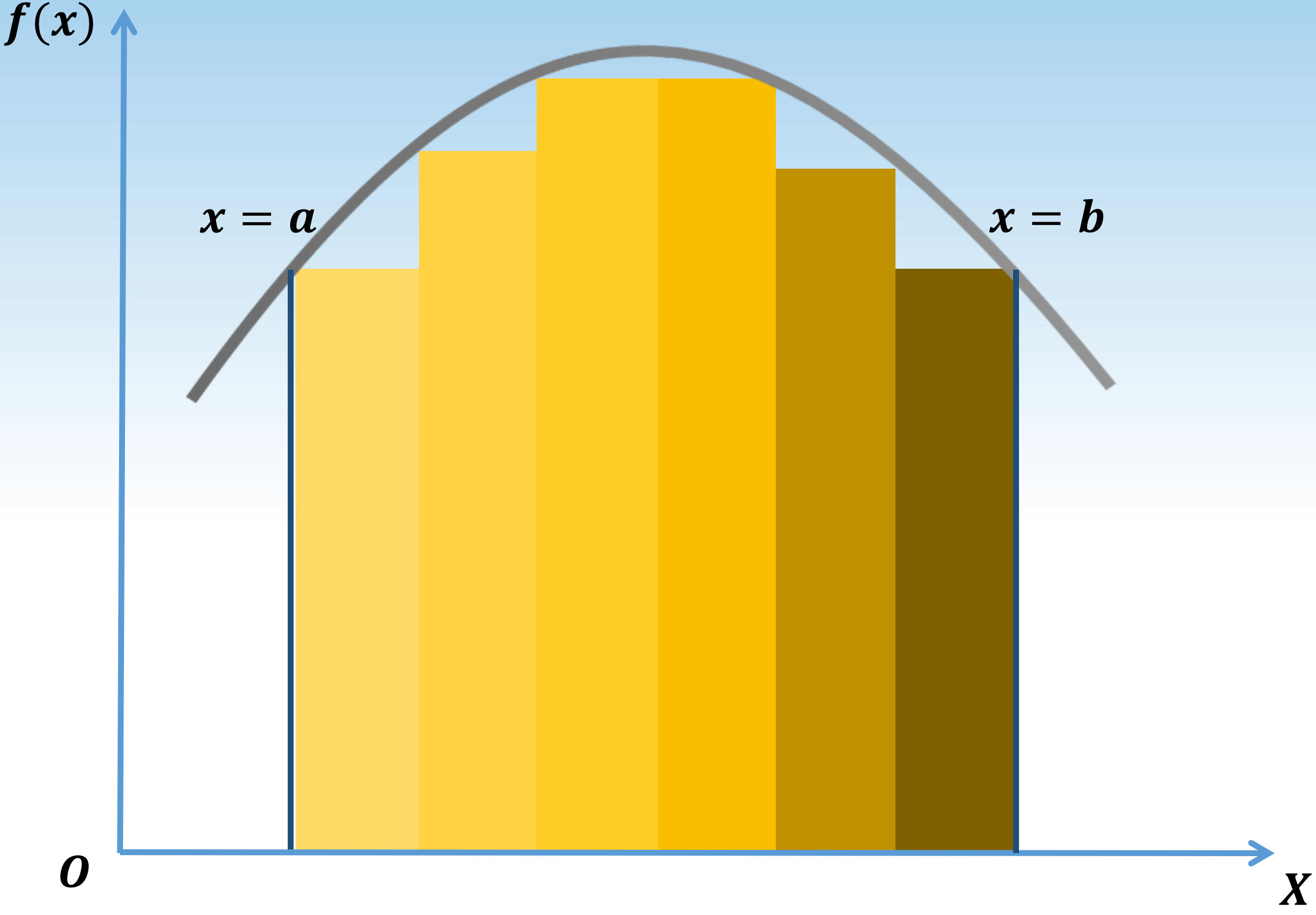
$$= \frac{1}{2}[e-1]$$

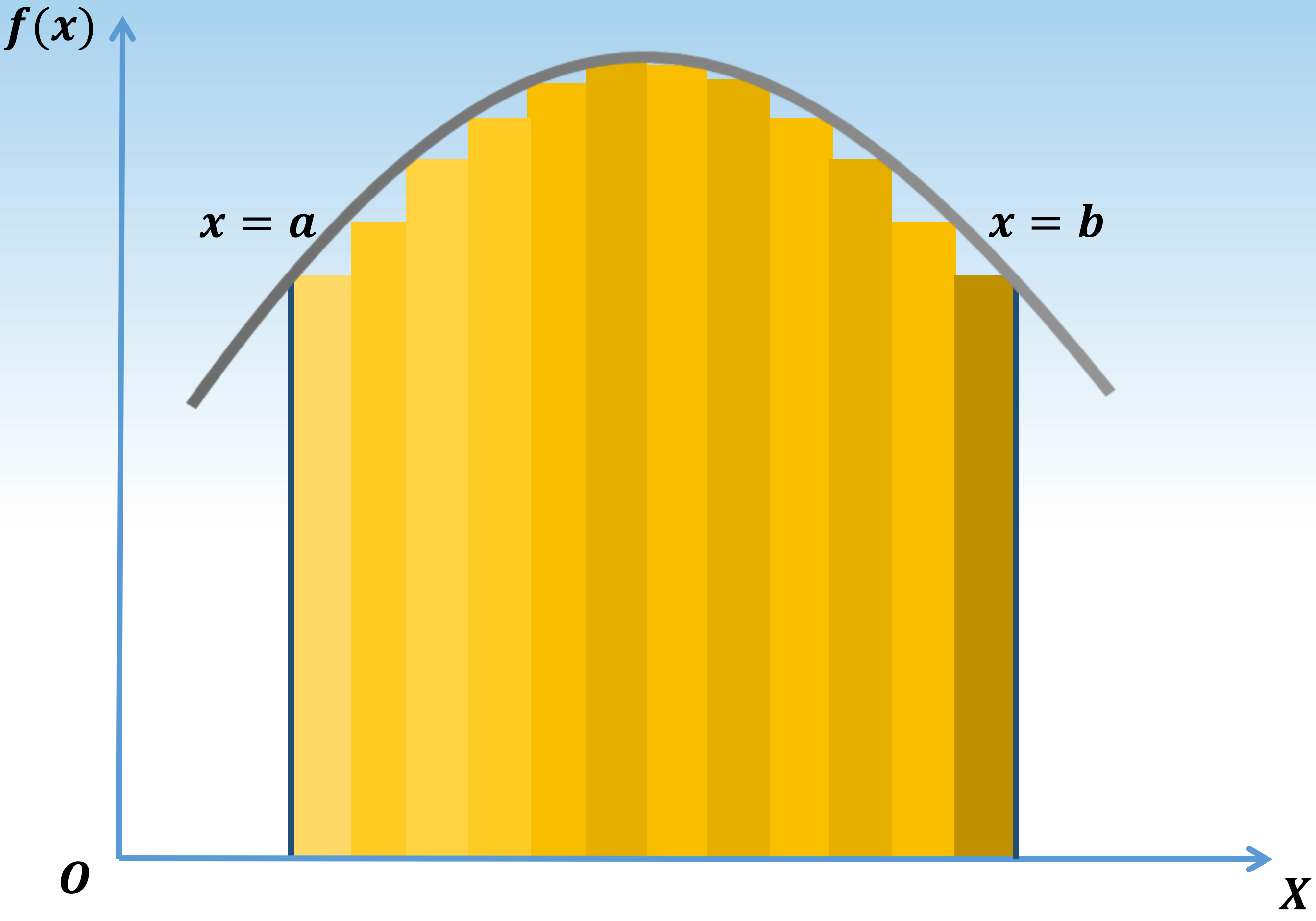


$f(x)$
$x = a$
$x = b$
O
X

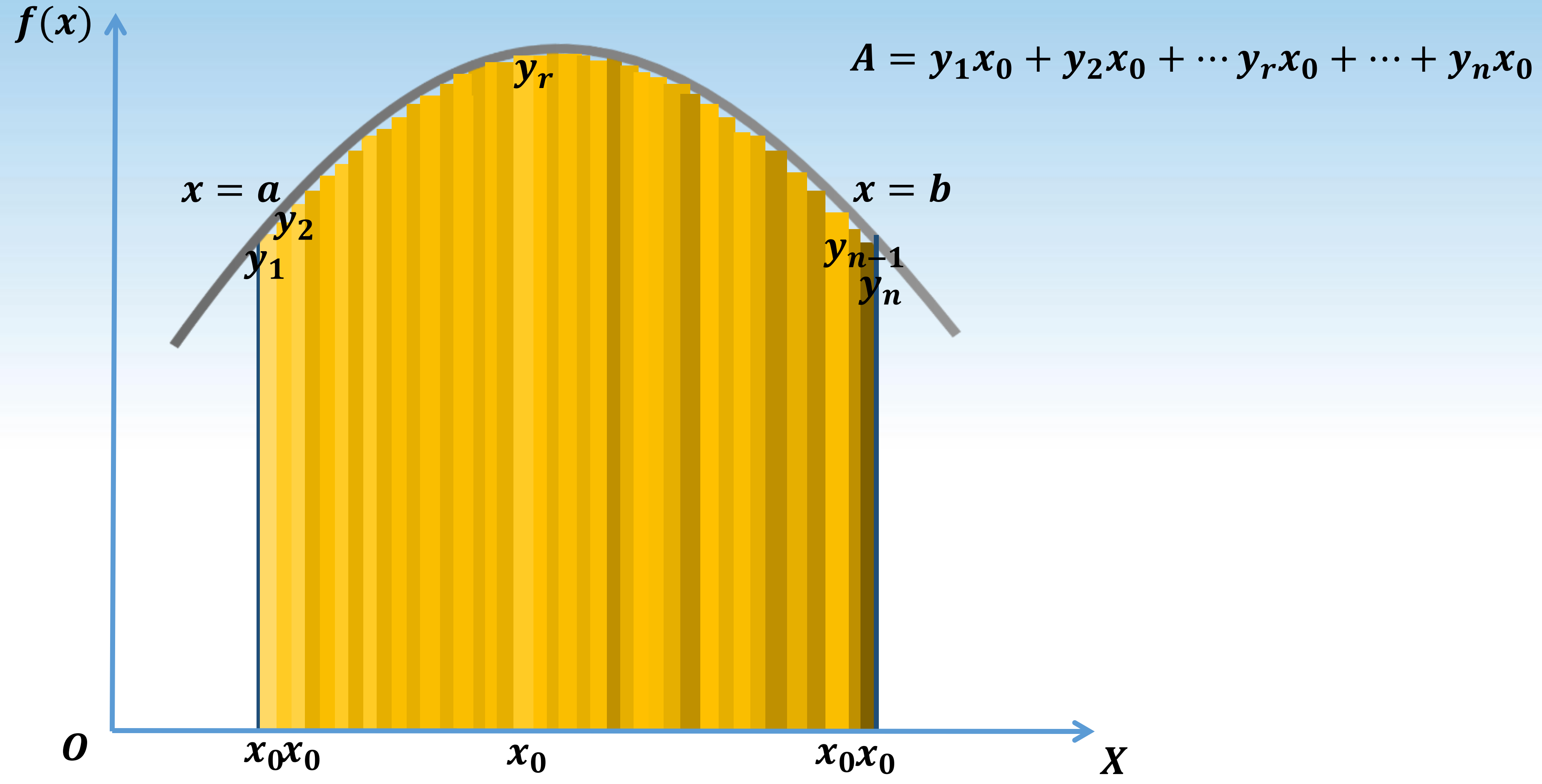

$f(x)$
$x = a$
$x = b$
O
X

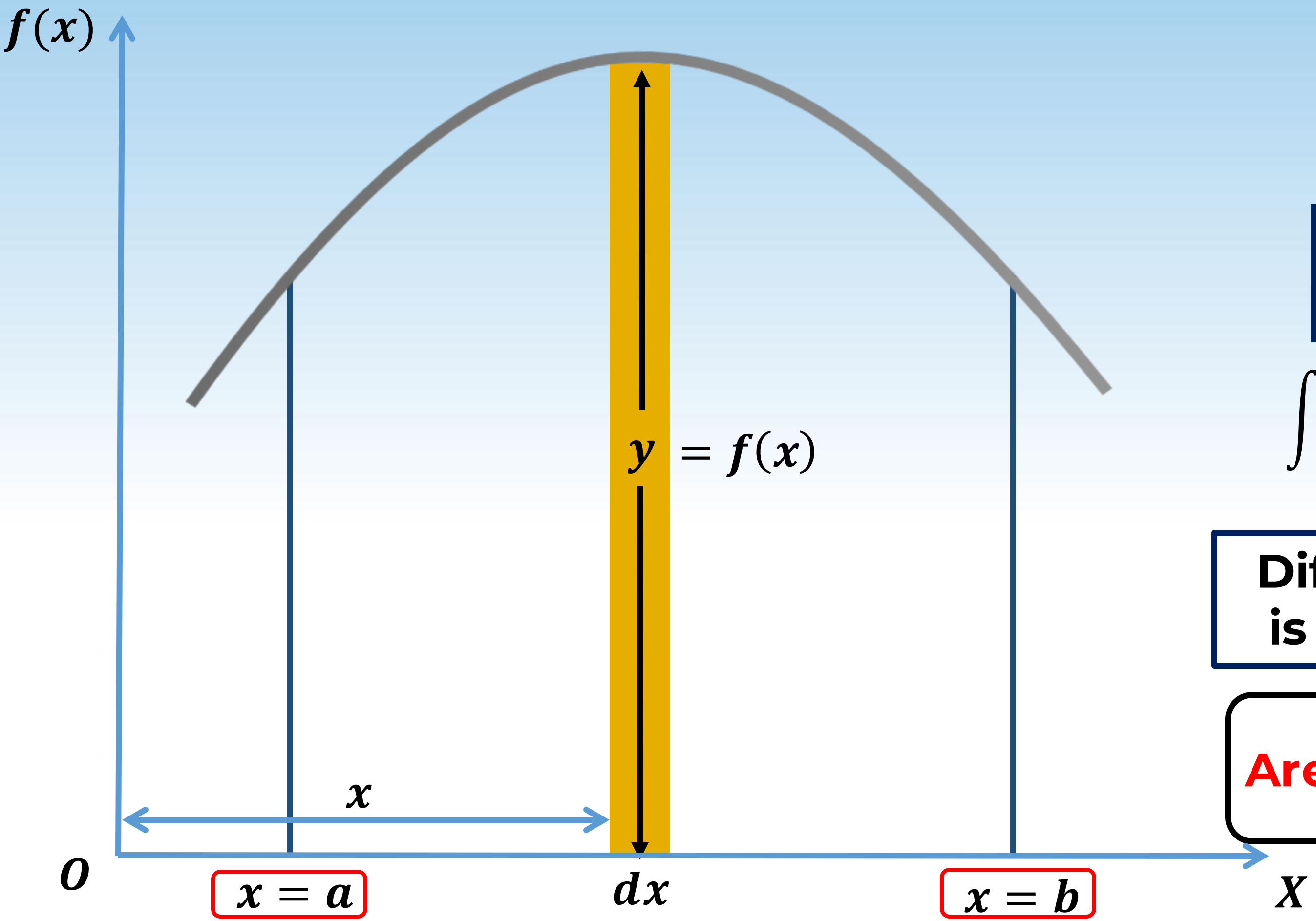

$f(x)$
$x = a$
$x = b$
O
X


$f(x)$
$x = a$
$x = b$
O
X



$f(x)$
$x = a$
$x = b$
O
X


$f(x)$
$A = y_1x_0 + y_2x_0 + \cdots y_rx_0 + \cdots + y_nx_0$
$x = a$
$x = b$
y_1
y_2
y_r
y_{n-1}
y_n
O
x_0x_0
x_0
x_0x_0
X



$$dA = y\,dx$$

$$dA = f(x)dx$$

$$\boxed{\frac{dA}{dx} = f(x)}$$

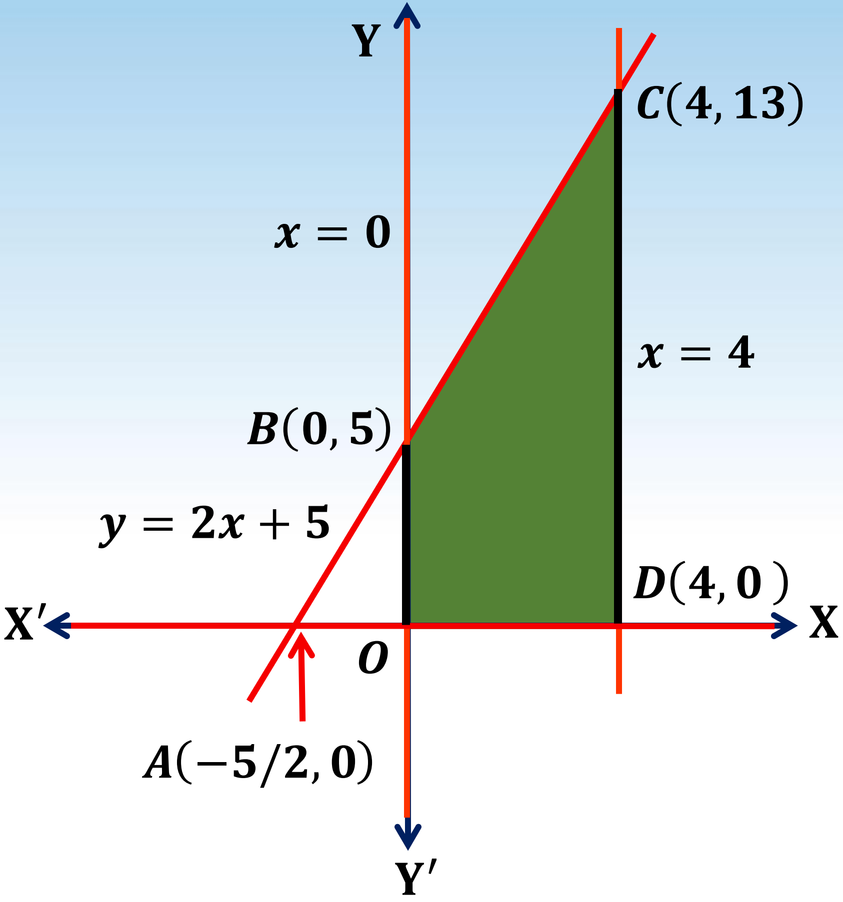
$$\int f(x)\,dx = A$$

Diff. of A w.r.t. x is equal to $f(x)$

Area $= \displaystyle\int_a^b f(x)\,dx$

Q. Find the area of the region bounded by $y = 2x + 5$ and x-axis for $x = 0$ to $x = 4$ in first quadrant.

Sol.



Q. Find the area of the region bounded by $y = 2x + 5$ and x-axis for $x = 0$ to $x = 4$ in first quadrant.

Sol. $$\boxed{Area = \int_a^b f(x)\,dx}$$

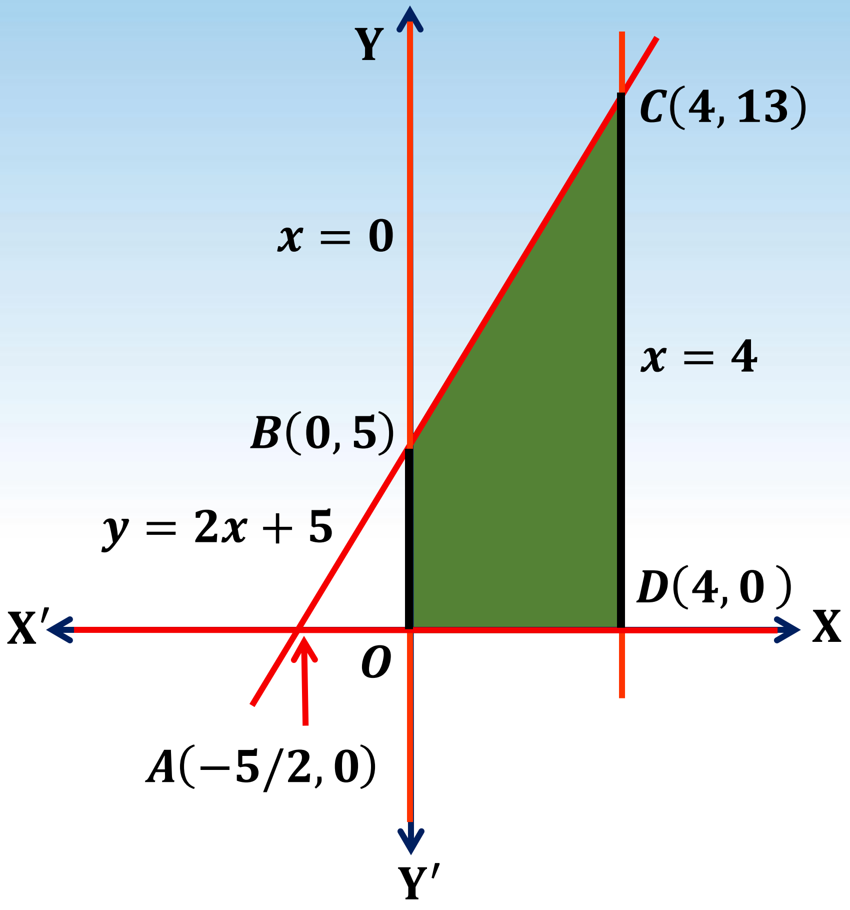
$f(x) = y = 2x + 5 \qquad a = 0 \qquad b = 4$

$$Area\ of\ BCDOB = \int_0^4 (2x + 5)\,dx$$

$$= \left[x^2 + 5x\right]_0^4$$

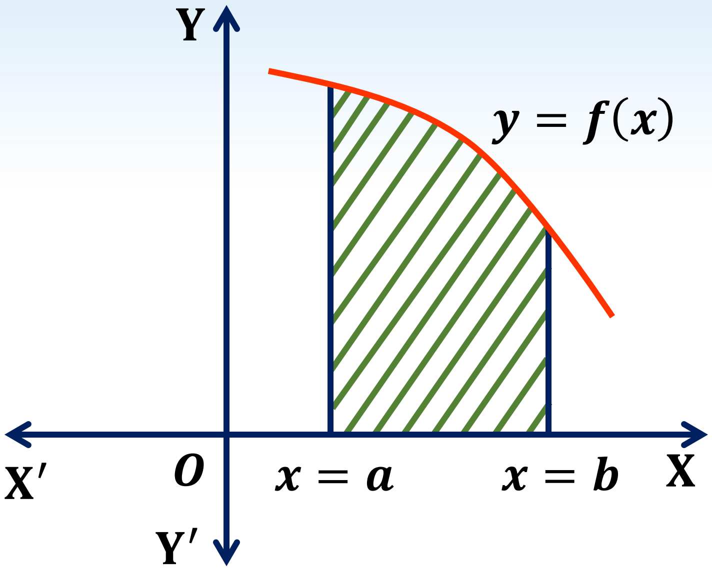
$$= \left[(4)^2 + 5(4)\right] - \left[(0)^2 + 5(0)\right]$$

$= $ **36 units**



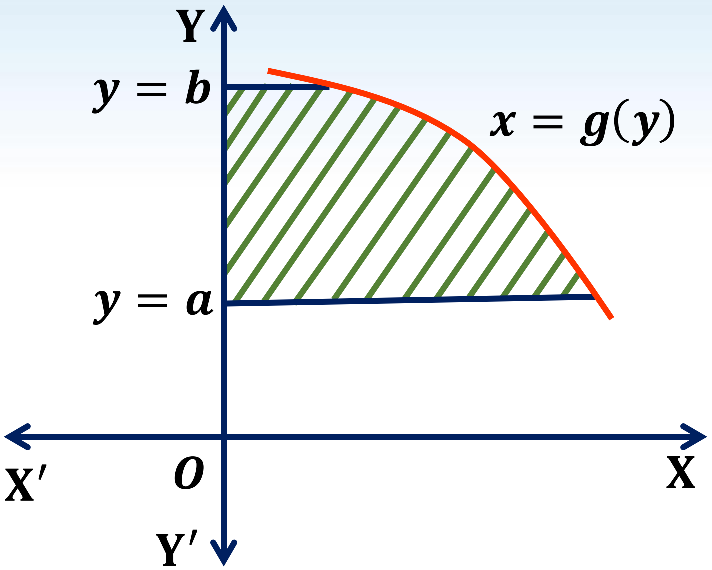
$$A = \int_a^b f(x)\,dx$$

The area of the region bounded by the curve $y = f(x)$, x-axis and the lines $x = a$ and $x = b$ $(b > a)$.



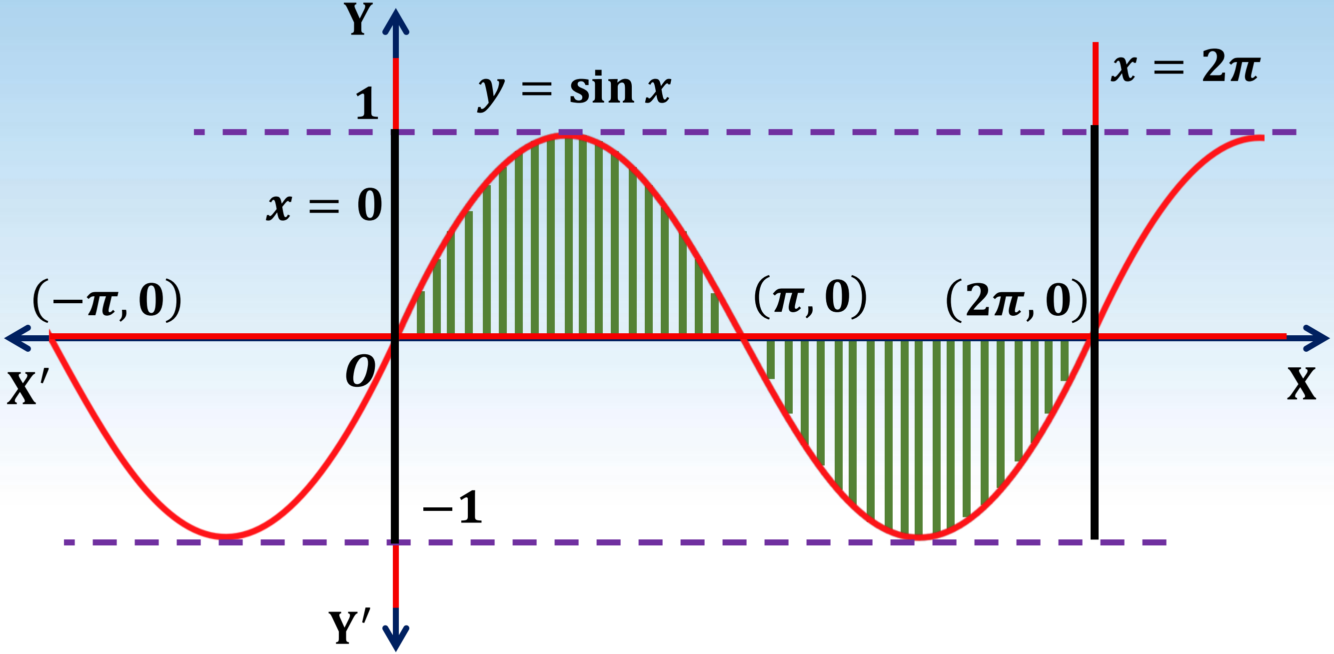
$$A = \int_a^b g(y)\,dy$$

The area of the region bounded by the curve $x = g(y)$, y-axis and the lines $y = a$ and $y = b(b > a)$.



Q. Find the area of the region bounded by $y = \sin x$ and x-axis for $x = 0$ to $x = 2\pi$.

Sol.



Q. Find the area of the region bounded by $y = \sin x$ and x-axis for $x = 0$ to $x = 2\pi$.

Sol. $$\boxed{A = \int_a^b f(x)\,dx}$$

$$f(x) = y = \sin x$$

$$a = 0 \qquad b = 2\pi$$

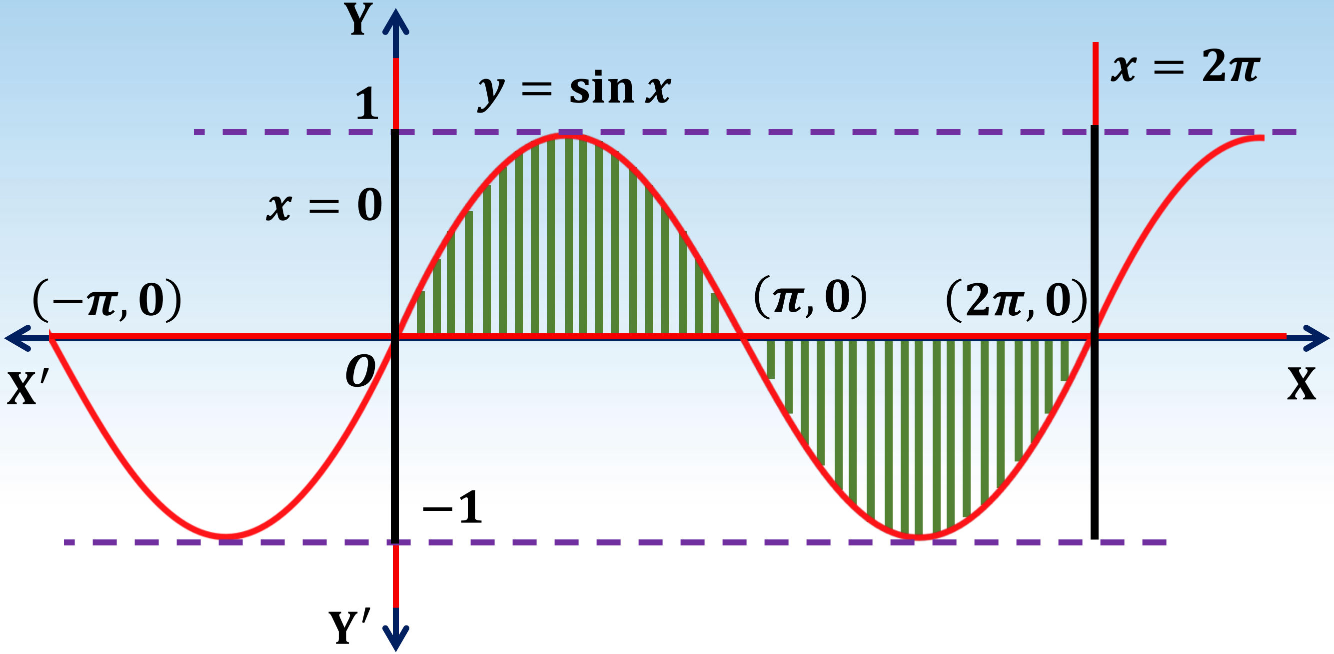
$$A = \int_0^{2\pi} \sin x\,dx$$

$$= [-\cos x]_0^{2\pi}$$

$$= (-\cos 2\pi) - (-\cos 0)$$

$$= (-1) - (-1)$$

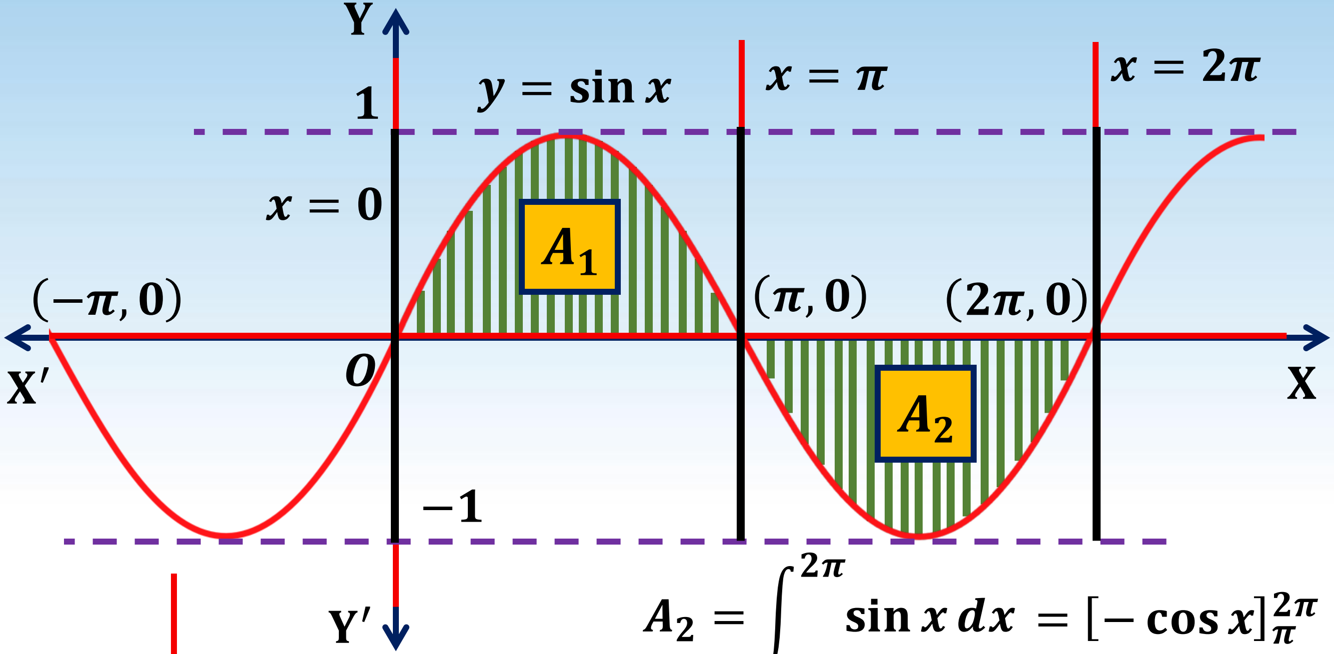
$$= 0$$



Q. Find the area of the region bounded by $y = \sin x$ and x-axis for $x = 0$ to $x = 2\pi$.

Sol.

$$\boxed{A = \int_a^b f(x)\,dx}$$



$f(x) = y = \sin x$

$a = 0 \qquad b = \pi$

$$A_1 = \int_0^{\pi} \sin x\,dx$$

$$= [-\cos x]_0^{\pi}$$

$$= (-\cos \pi) - (-\cos 0)$$

$$= \big(-(-1)\big) - (-1)$$

$= 2$ Units

$f(x) = y = \sin x$

$a = \pi \qquad b = 2\pi$

$$A_2 = \int_{\pi}^{2\pi} \sin x\,dx = [-\cos x]_{\pi}^{2\pi}$$

$$= (-\cos 2\pi) - (-\cos \pi)$$

$$= \big(-(1)\big) - \big(-(-1)\big)$$

$= -2$ Units

Q. Find the area of the region bounded by $y = \sin x$ and x-axis for $x = 0$ to $x = 2\pi$.

Sol. $$A = \int_a^b f(x)\,dx$$

$A_1 = $ 2 Units

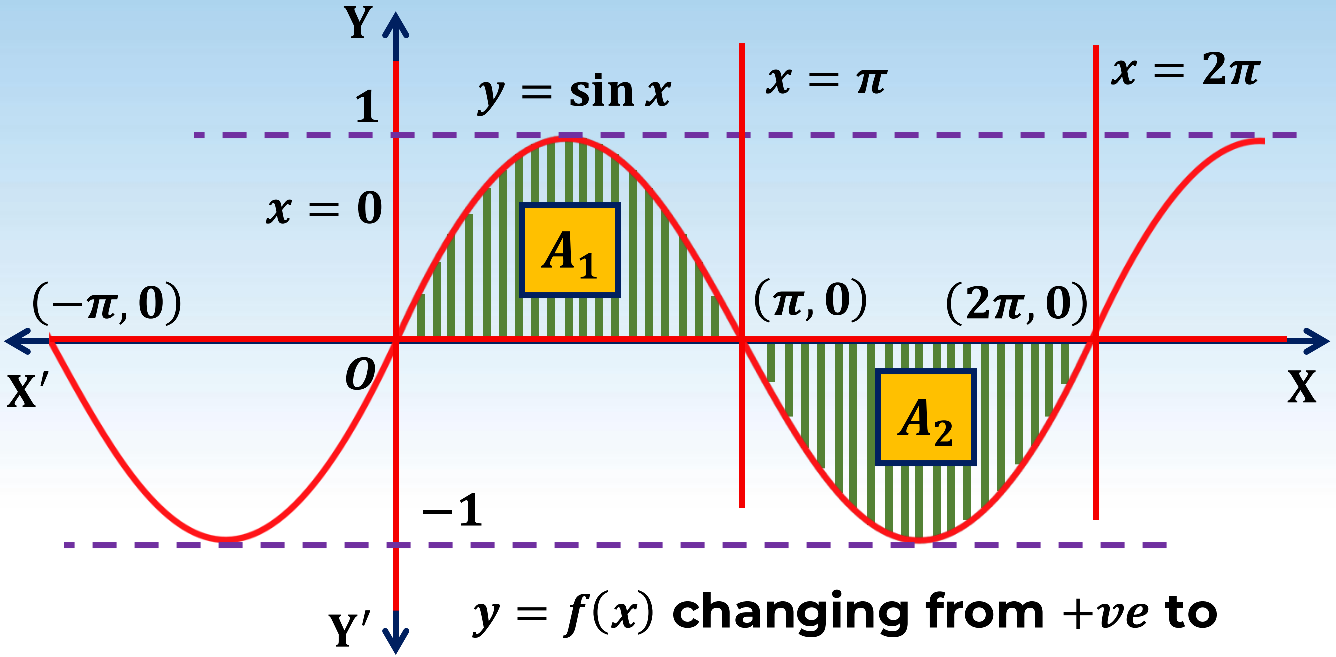
$A_2 = -$ 2 Units

$|A_2| = $ 2 Units

$A = A_1 + |A_2|$

$= $ 4 Units

$$A = \left|\int_a^b f(x)\,dx\right|$$



$y = f(x)$ changing from $+ve$ to $-ve$ at $x = c$, where $c \in (a, b)$, then

$$A = \left|\int_a^c f(x)\,dx\right| + \left|\int_c^b f(x)\,dx\right|$$

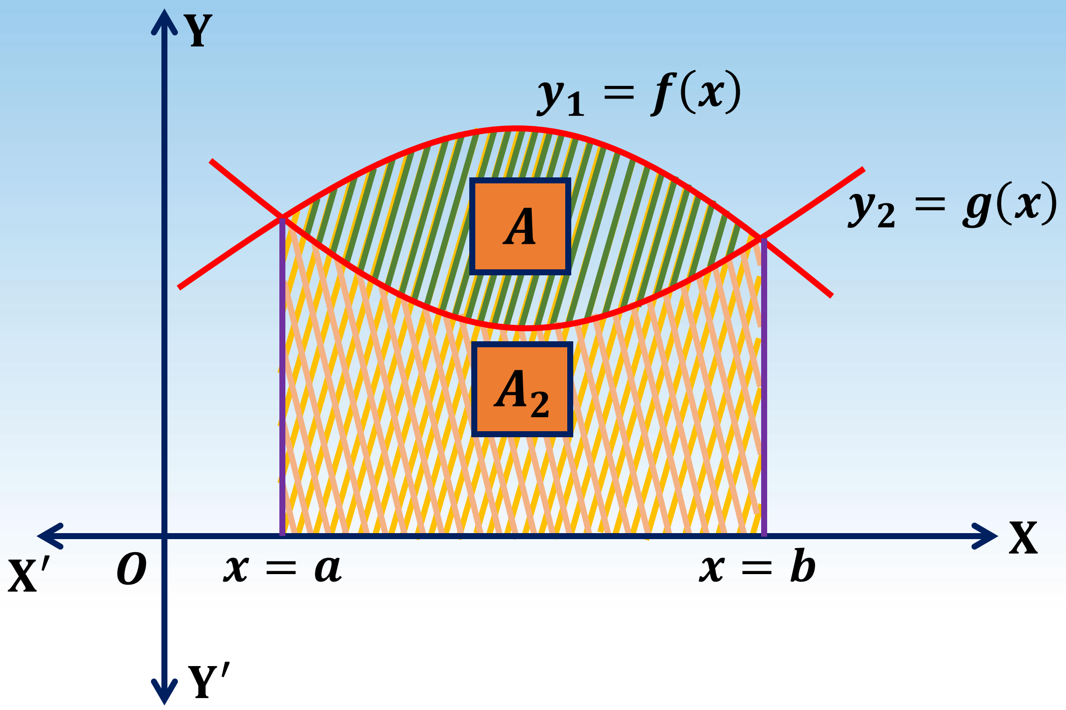
Summary

The area of the region bounded by the **curve** $y = f(x), x$**-axis** and the **lines** $x = a$ and $x = b\ (b > a)$ is given by the formula:

$$Area = \left|\int_a^b f(x)\,dx\right|$$

The area of the region bounded by the curve $x = g(y), y$**-axis** and the **lines** $y = c$ and $y = d(d > c)$ is given by the formula

$$Area = \left|\int_c^d g(y)\,dy\right|$$



$$A_1 = \left| \int_a^b f(x)\,dx \right|$$

$$A_2 = \left| \int_a^b g(x)\,dx \right|$$

$$A = A_1 - A_2$$

$$A = \left| \int_a^b f(x)\,dx \right| - \left| \int_a^b g(x)\,dx \right|$$

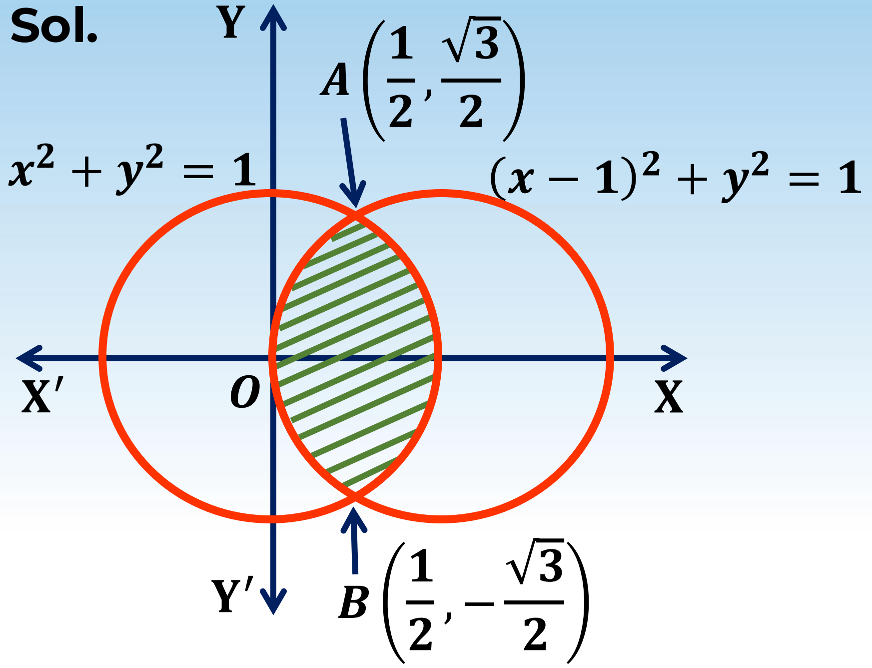
$$A = \left| \int_a^b f(x)\,dx - \int_a^b g(x)\,dx \right|$$

$$\boxed{A = \left| \int_a^b (Upper\ Curve - Lower\ Curve)\,dx \right|}$$

$$\boxed{A = \left| \int_a^b \big(f(x) - g(x)\big)\,dx \right|}$$

Q. Find the area bounded by curves $(x-1)^2+y^2=1$ and $x^2+y^2=1$.

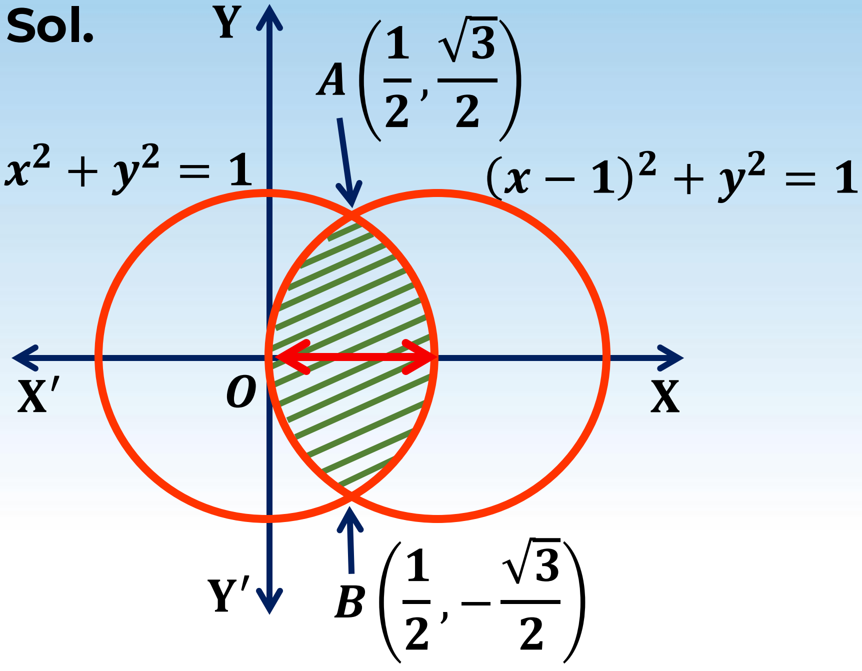
Sol.



$$\boxed{A=\left|\int_a^b \left(f(y)-g(y)\right)dy\right|}$$

Q. Find the area bounded by curves $(x-1)^2+y^2=1$ and $x^2+y^2=1$.

Sol.



$$\boxed{A=\left|\int_a^b \left(f(y)-g(y)\right)dy\right|}$$

$f(y)=\sqrt{1-y^2}$ $\quad g(y)=1-\sqrt{1-y^2}$

$$x^2+y^2=1$$

$$\Rightarrow x^2=1-y^2$$

$$\Rightarrow x=\pm\sqrt{1-y^2}$$

$$(x-1)^2+y^2=1$$

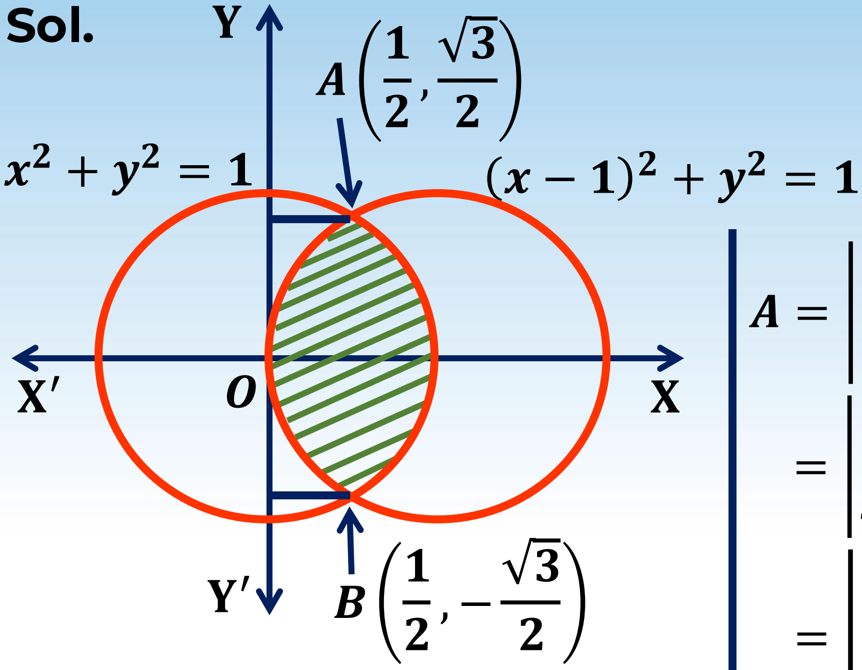
$$\Rightarrow (x-1)^2=1-y^2$$

$$\Rightarrow x-1=\pm\sqrt{1-y^2}$$

$$\Rightarrow x=1\pm\sqrt{1-y^2}$$

Q. Find the area bounded by curves $(x-1)^2+y^2=1$ and $x^2+y^2=1$.

Sol.



$$f(y)=\sqrt{1-y^2} \quad g(y)=1-\sqrt{1-y^2}$$

$$a=-\frac{\sqrt{3}}{2} \text{ and } b=\frac{\sqrt{3}}{2}$$

$$\boxed{A=\left|\int_a^b (f(y)-g(y))\,dy\right|}$$

$$A=\left|\int_{-\sqrt{3}/2}^{\sqrt{3}/2}\left(\sqrt{1-y^2}-\left(1-\sqrt{1-y^2}\right)\right)dx\right|$$

$$=\left|\int_{-\sqrt{3}/2}^{\sqrt{3}/2}\left(2\sqrt{1-y^2}-1\right)dx\right|$$

$$=\left|\left[2\left(\frac{y}{2}\sqrt{1-y^2}+\frac{1}{2}\sin^{-1}y\right)-y\right]_{-\sqrt{3}/2}^{\sqrt{3}/2}\right|$$

$$\boxed{A=\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)\text{ Units}}$$

Summary

The area of the region bounded by the curve $y = f(x), y = g(x)$ and the lines $x = a$ and $x = b$ $(b > a)$ is given by the formula:

$$Area = \left|\int_a^b \big(f(x) - g(x)\big)\,dx\right|$$

where $f(x) \geq g(x)$ in $[a, b]$

If $f(x) \geq g(x)$ in $[a, c]$ and $f(x) \leq g(x)$ in $[c, b], a < c < b$, then

$$Area = \left|\int_a^c \big(f(x) - g(x)\big)\,dx\right| + \left|\int_c^b \big(g(x) - f(x)\big)\,dx\right|$$