VIGNAN'S

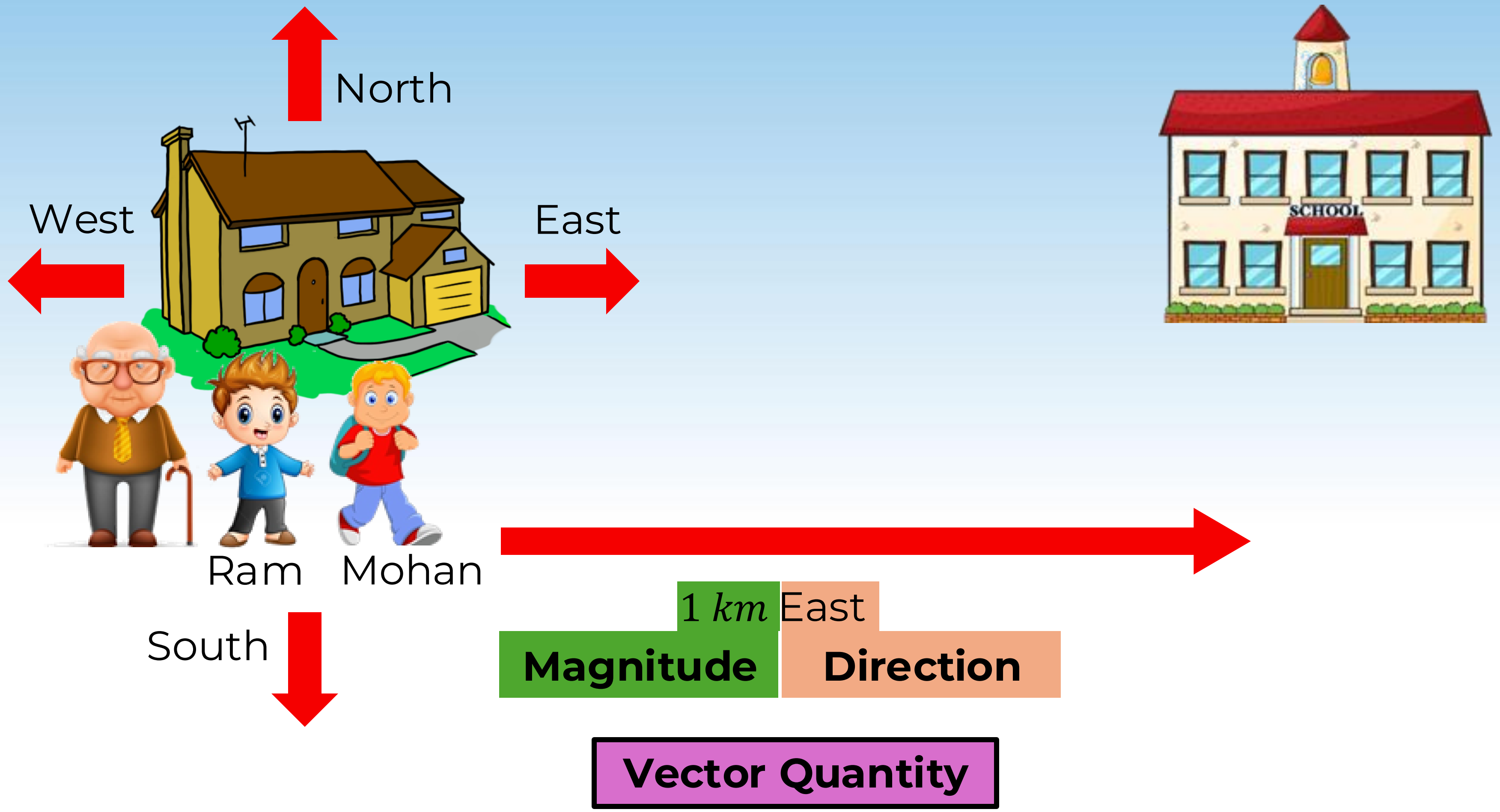
FOUNDATION FOR SCIENCE, TECHNOLOGY & RESEARCH

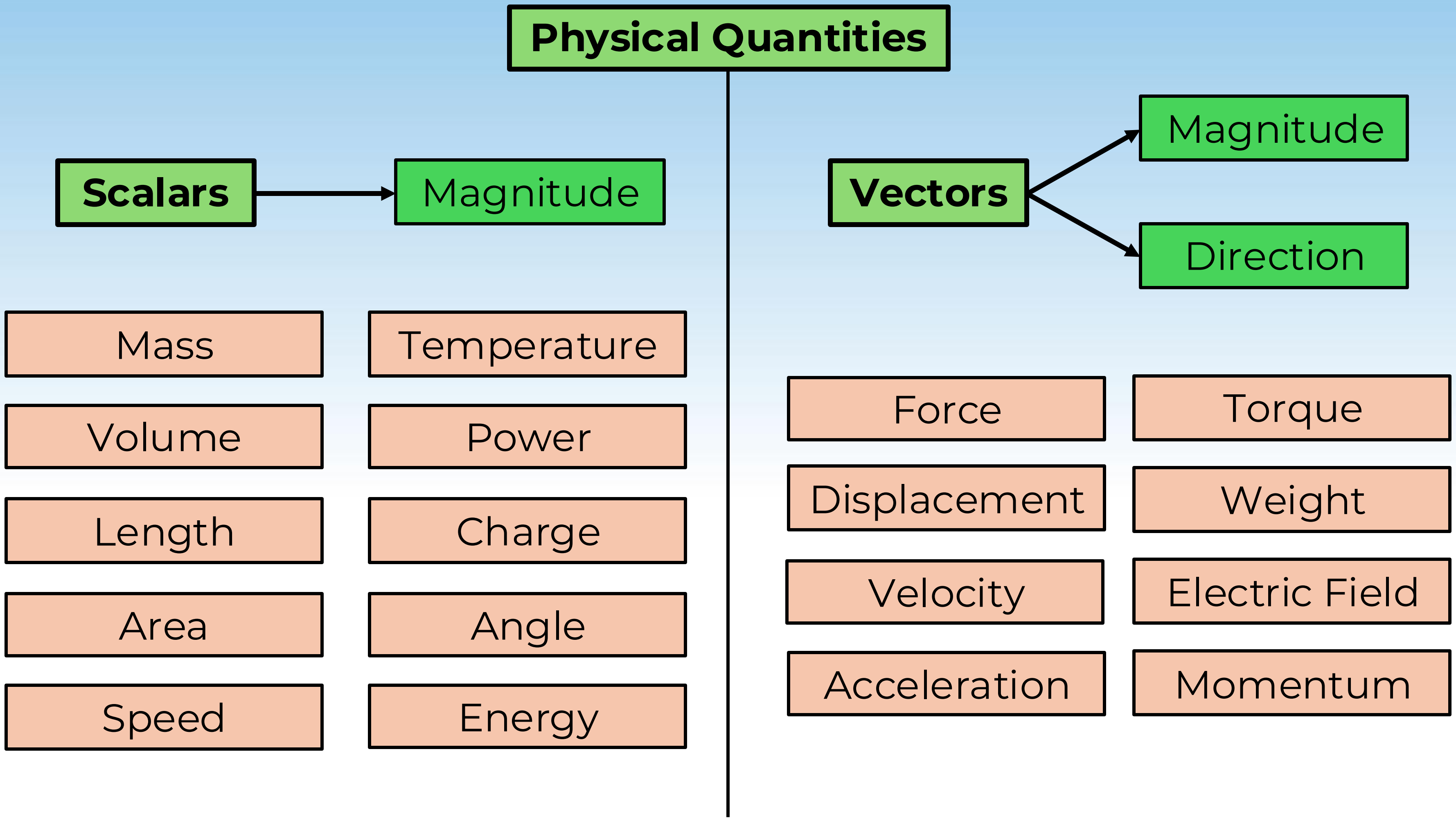
(Deemed to be University) - Estd. u/s 3 of UGC Act 1956

Credits: Avanti Sankalp Program

Unit 5: Vector Algebra

D Bhanu Prakash


North
West
East
SCHOOL
Ram
Mohan
South
1 km East
Magnitude
Direction
Vector Quantity


Physical Quantities
Scalars
Magnitude
Vectors
Magnitude
Direction
Mass
Temperature
Volume
Power
Length
Charge
Area
Angle
Speed
Energy
Force
Torque
Displacement
Weight
Velocity
Electric Field
Acceleration
Momentum

Q. Classify the following measures as scalars and vectors.

Sol.

(i) $\boxed{13\ kg}$ $\boxed{\rightarrow \text{Scalar}}$

(ii) $\boxed{2\ meters\ \text{south - west}}$ $\boxed{\rightarrow \text{Vector}}$

(iii) $\boxed{45°}$ $\boxed{\rightarrow \text{Scalar}}$

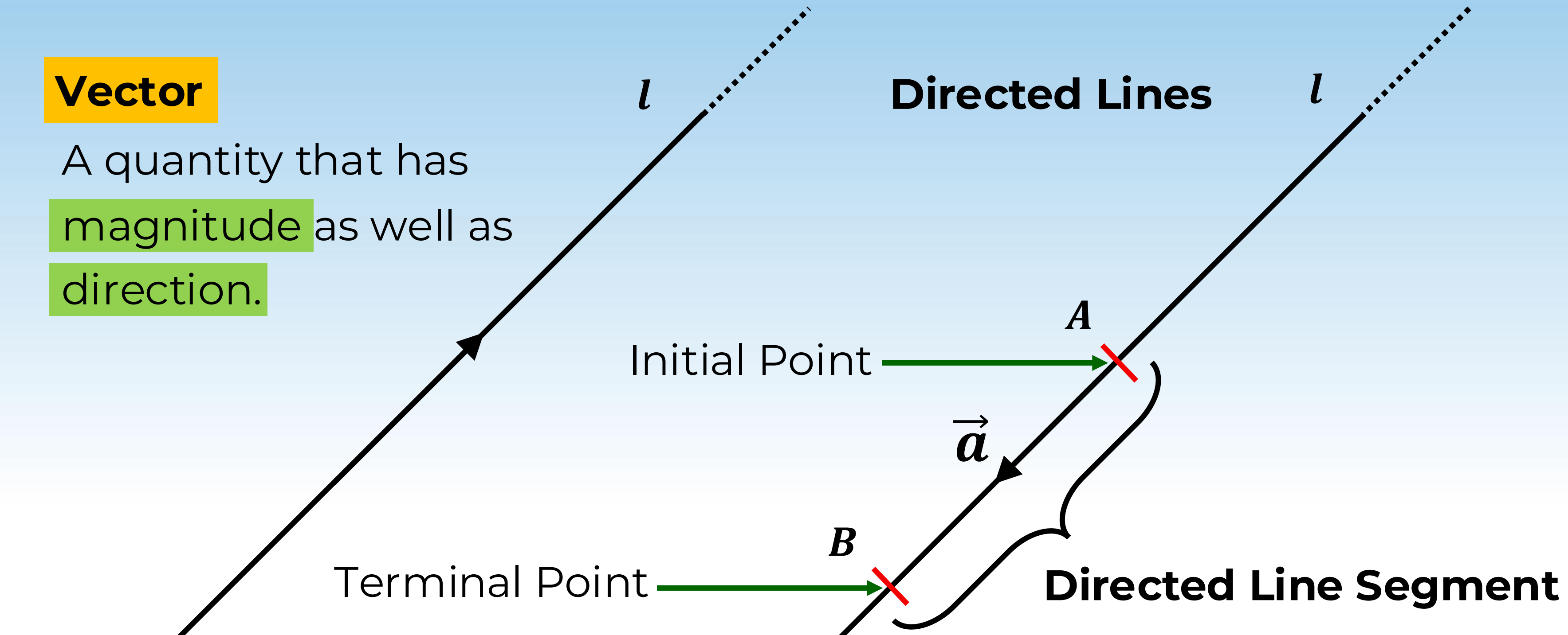
(iv) $\boxed{4\ joule}$ $\boxed{\rightarrow \text{Scalar}}$

(v) $\boxed{10^{-21}\ coulomb}$ $\boxed{\rightarrow \text{Scalar}}$

(vi) $\boxed{30\ m/s^2}$ $\boxed{\rightarrow \text{Vector}}$

Vector

A quantity that has magnitude as well as direction.



$$\boxed{\overrightarrow{AB} = \vec{a}}$$

Magnitude : $\boxed{|\overrightarrow{AB}| = a}$

Position Vector

$$\overrightarrow{OP} = \vec{r}$$

In $\triangle\, QOR$

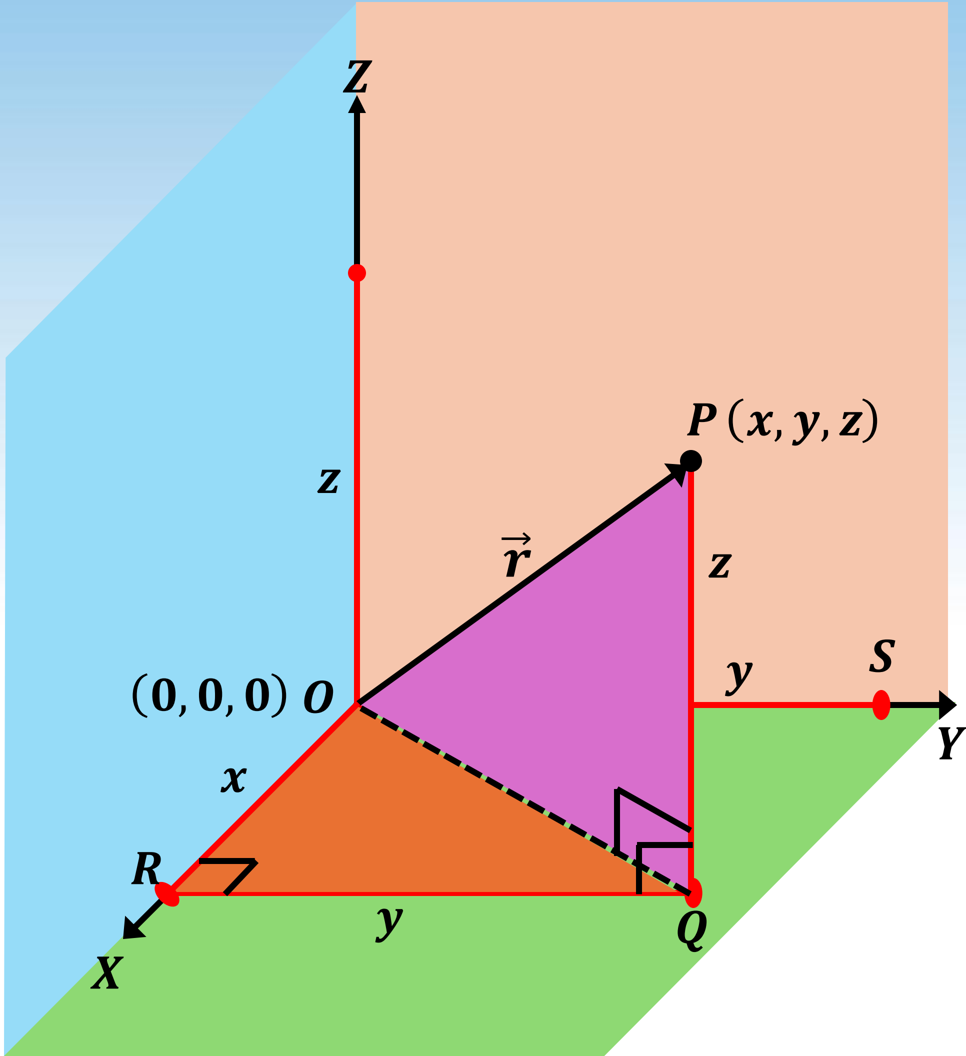
$$OQ^2 = OR^2 + RQ^2$$

$$\boxed{OQ^2 = x^2 + y^2}$$

In $\triangle\, POQ$

$$OP = \sqrt{OQ^2 + PQ^2}$$

$$\boxed{\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2 + z^2} = |\vec{r}| = r}$$



Position Vector

$\overrightarrow{OA} = \vec{a}$

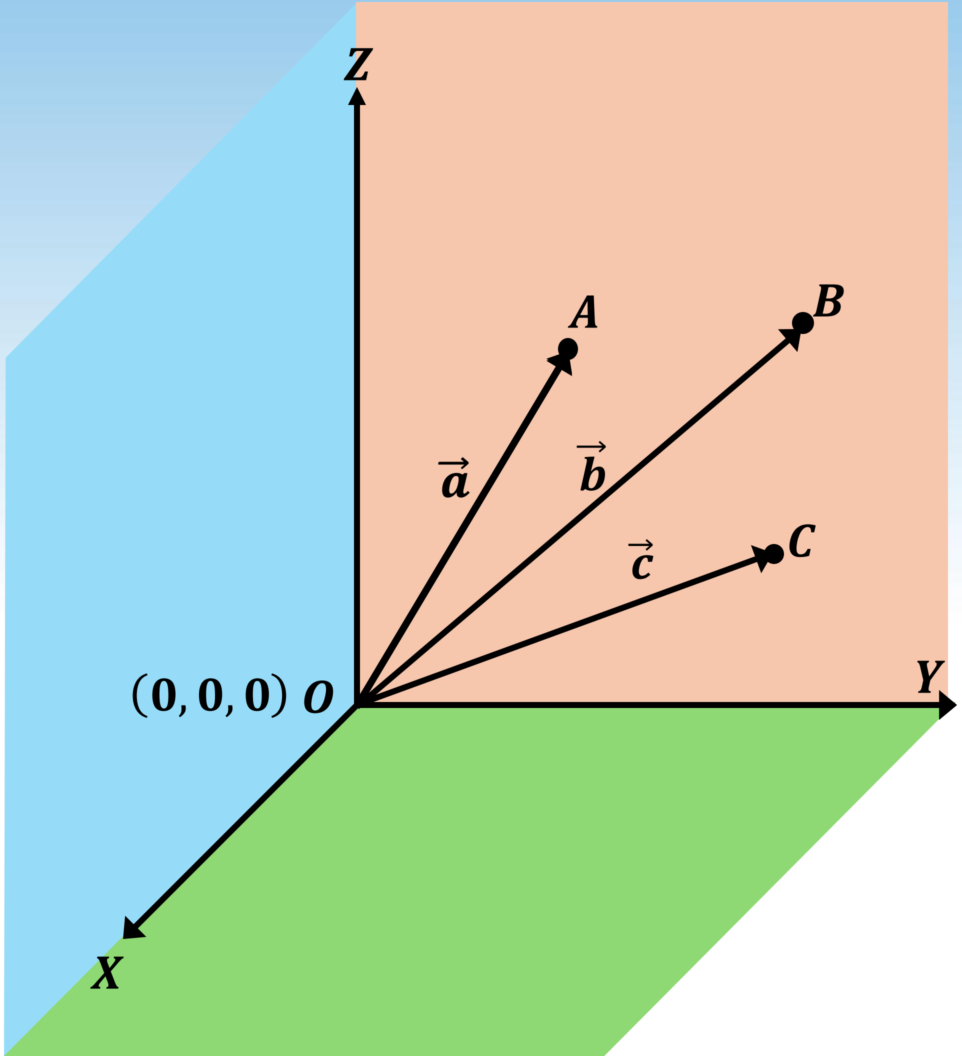
$|\overrightarrow{OA}| = a$

$\overrightarrow{OB} = \vec{b}$

$|\overrightarrow{OB}| = b$

$\overrightarrow{OC} = \vec{c}$

$|\overrightarrow{OC}| = c$



Types of Vectors

Zero/Null Vector

Unit Vector

Coinitial Vectors

Collinear Vectors

Equal Vectors

Types of Vectors

Zero/Null Vector

Unit Vector

Coinitial Vectors

Collinear Vectors

Equal Vectors

A vector whose initial and terminal points coincide.

Ex. $\overrightarrow{AA}, \overrightarrow{BB}$ etc.

$$\left|\overrightarrow{AA}\right| = 0$$

$$\left|\overrightarrow{BB}\right| = 0$$

Types of Vectors

Zero/Null Vector

Unit Vector

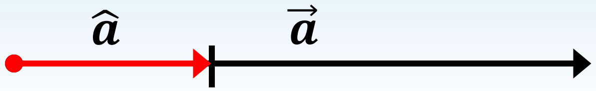
Coinitial Vectors

Collinear Vectors

Equal Vectors

A vector whose magnitude is unity.

Unit vector in the direction of $\vec{a}$ is $\hat{a}$.



$$\boxed{\hat{a} = \frac{\vec{a}}{|a|}}$$

$$\boxed{|\hat{a}| = 1}$$

Types of Vectors

Zero/Null Vector

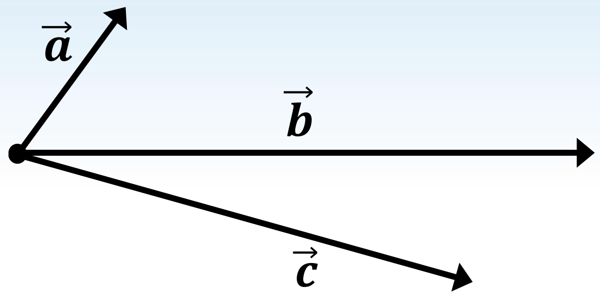
Unit Vector

Coinitial Vectors

Collinear Vectors

Equal Vectors

Two or more vectors having the same initial points.

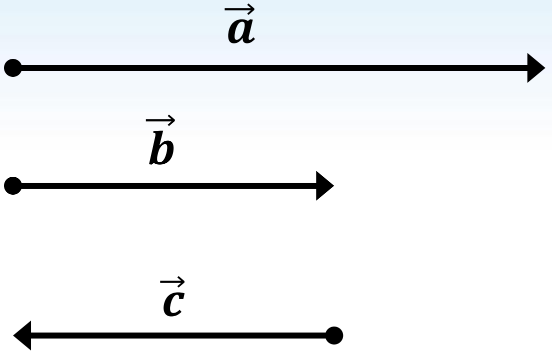


$\vec{a}$,$\vec{b}$ and $\vec{c}$ are coinitial vectors.

Types of Vectors

- Zero/Null Vector
- Unit Vector
- Coinitial Vectors
- **Collinear Vectors**
- Equal Vectors

If two or more vectors are parallel to the same line, irrespective of their magnitude and directions.



$\vec{a}, \vec{b}$ and $\vec{c}$ are collinear vectors.

Types of Vectors

Zero/Null Vector

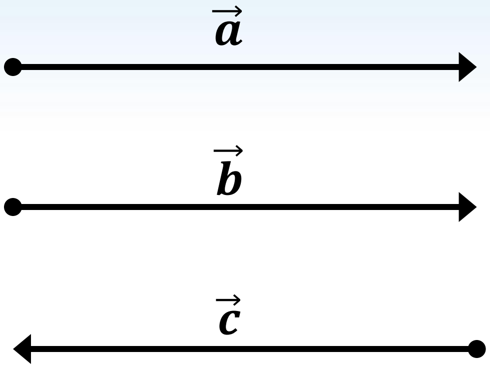
Unit Vector

Coinitial Vectors

Collinear Vectors

Equal Vectors

Two vectors having same magnitude and direction regardless of the positions of their initial points.



$\vec{a} = \vec{b}$

$\vec{a} \neq \vec{c}$

$\vec{b} \neq \vec{c}$

$\vec{a}, \vec{b}$ and $\vec{c}$ are collinear vectors.

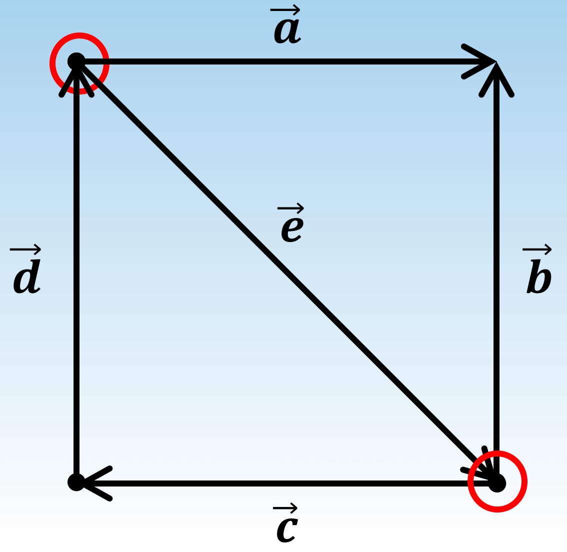
Q. In given figure (a square), identify the following vectors.

Sol.

(i) $\boxed{\text{Coinitial}}$

(ii) $\boxed{\text{Equal}}$

(iii) $\boxed{\text{Collinear but not equal}}$



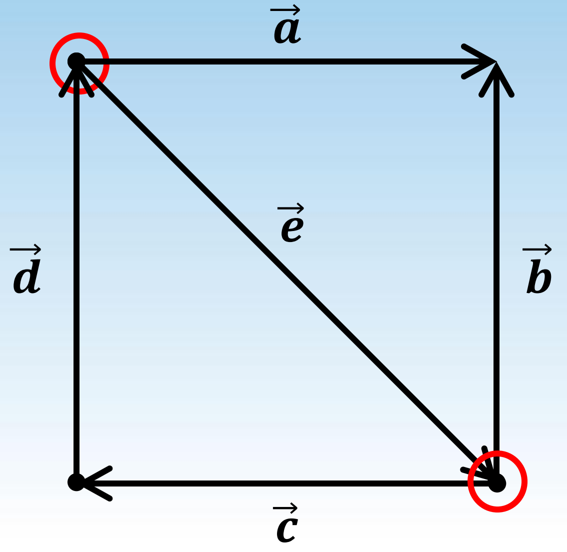
Q. In given figure (a square), identify the following vectors.

Sol.

(i) Coinitial → $\vec{a}$ and $\vec{e}$ | $\vec{c}$ and $\vec{b}$

(ii) Equal → $\vec{b}$ and $\vec{d}$

(iii) Collinear but not equal → $\vec{a}$ and $\vec{c}$



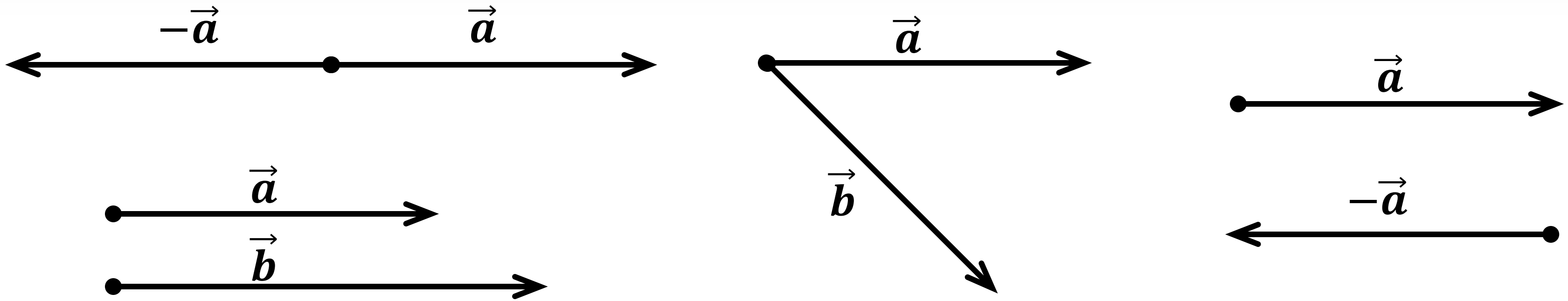
Q. Answer the following as true or false :

Sol. (i) $\boxed{\vec{a} \text{ and } -\vec{a} \text{ are collinear.}}$

(ii) $\boxed{\text{Two collinear vectors are always equal in magnitude.}}$

(iii) $\boxed{\text{Two vectors having same magnitude are collinear.}}$

(iv) $\boxed{\text{Two collinear vectors having the same magnitude are equal.}}$



Q. Answer the following as true or false :

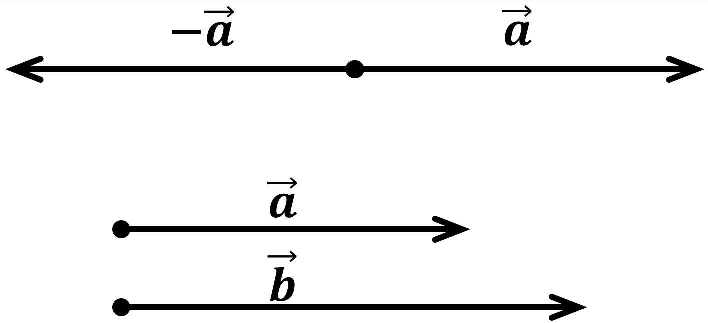
Sol.

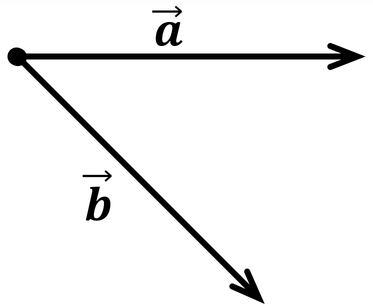
(i) $\vec{a}$ and $-\vec{a}$ are collinear.

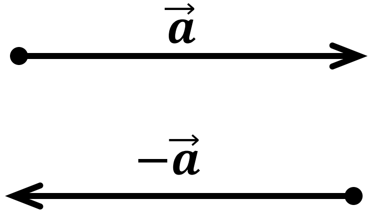
(ii) Two collinear vectors are always equal in magnitude.

(iii) Two vectors having same magnitude are collinear.

(iv) Two collinear vectors having the same magnitude are equal.







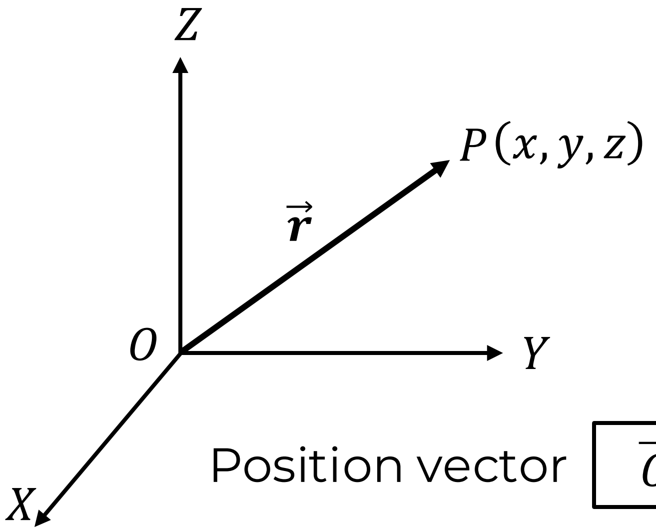
Summary

Scalar Quantity : A physical quantity that has only a **magnitude**.

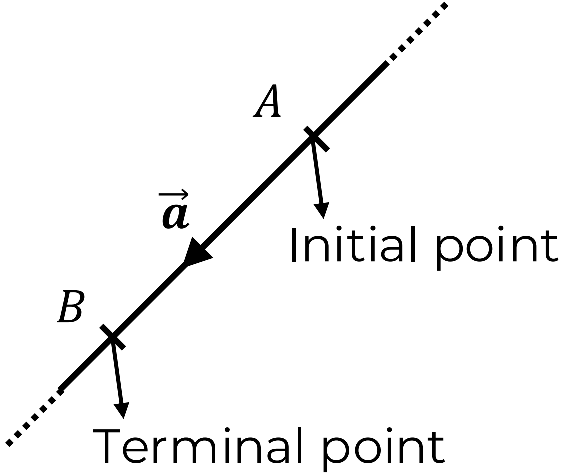
Vector Quantity : A physical quantity that has **magnitude** and **direction**.

Directed line segment $\boxed{\overrightarrow{AB} = \vec{a}}$

Magnitude $\boxed{|\overrightarrow{AB}| = a}$



Position vector $\boxed{\overrightarrow{OP} = \vec{r}}$



Summary

Types of Vectors

Zero vector → $\overrightarrow{AA} = \vec{0}$ → $|\overrightarrow{AA}| = 0$

Unit vector → $|\overrightarrow{AB}| = 1$

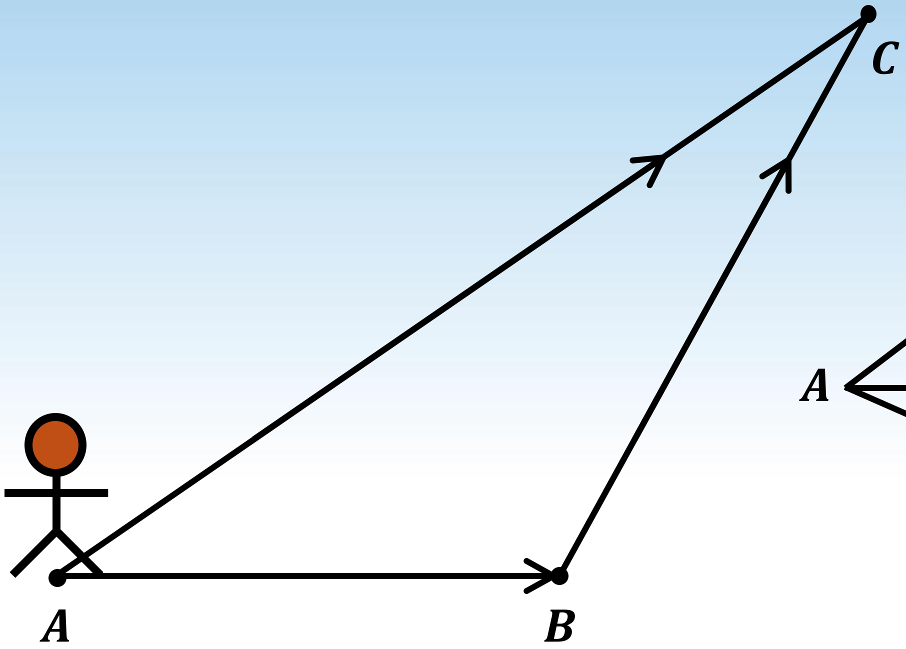
Coinitial vector → Vectors having same initial points.

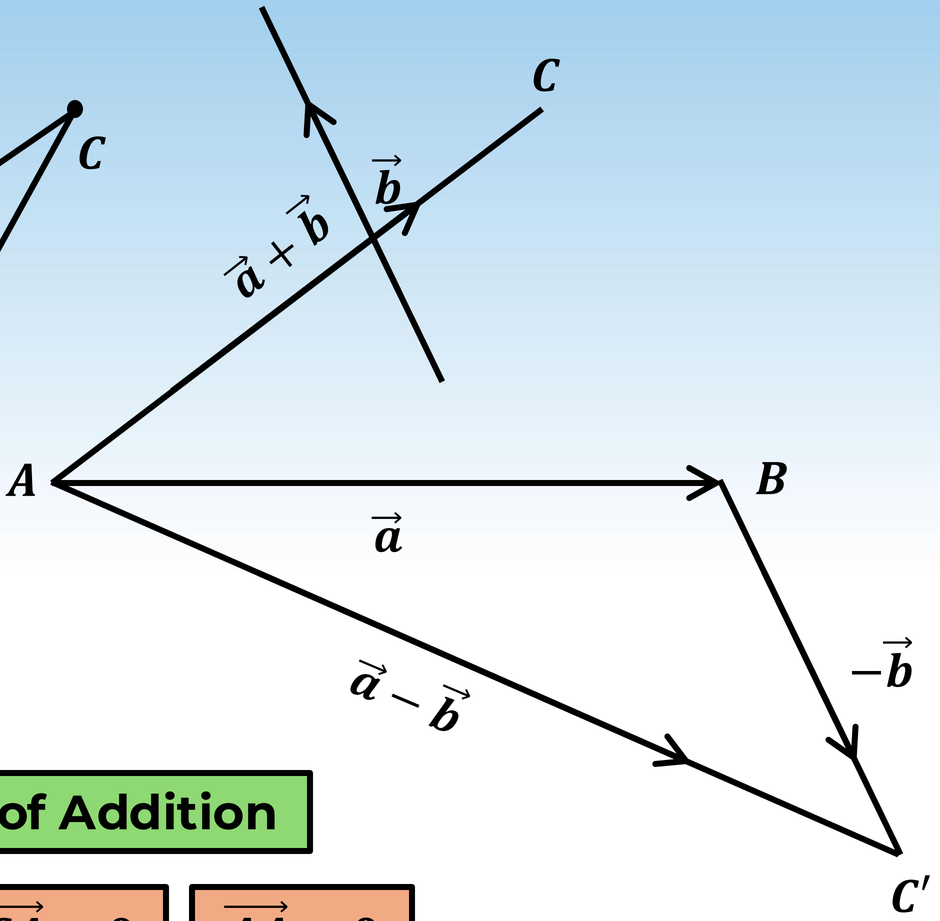
Collinear vector → Vectors parallel to the same line.

Equal vector → Vectors having same magnitude and direction.

Multiplication of a vector by a scalar → $\vec{a} \times \lambda = \lambda\vec{a}$ | $\lambda \rightarrow$ Scalar

Magnitude → $|\vec{a} \times \lambda| = |\lambda\vec{a}| = \lambda a$





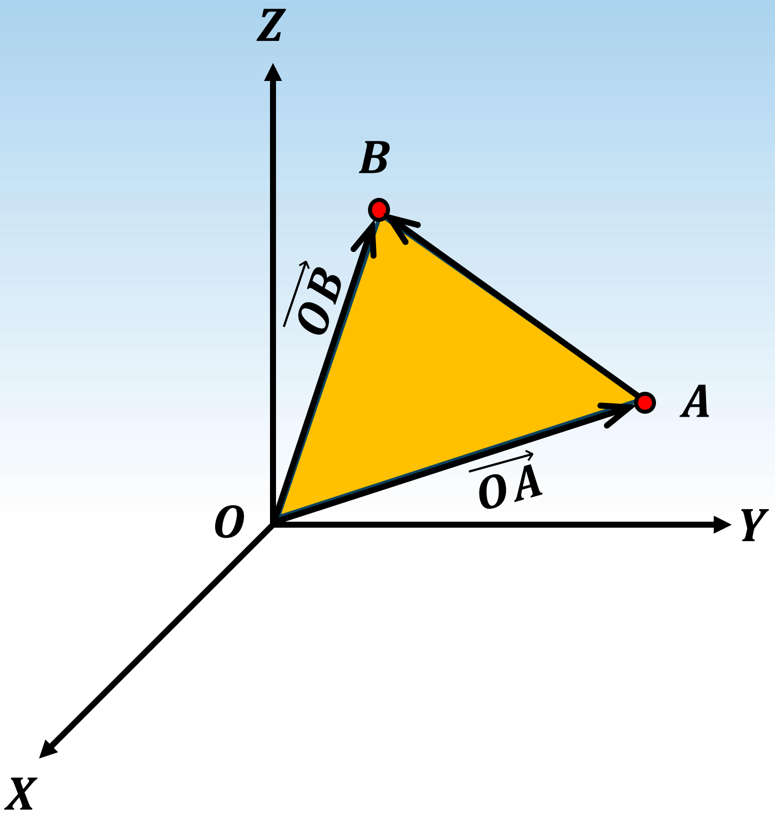
$\boxed{\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}}$

Triangle Law of Addition

$0 = \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC}$

$\boxed{\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0}$

$\boxed{\overrightarrow{AA} = 0}$

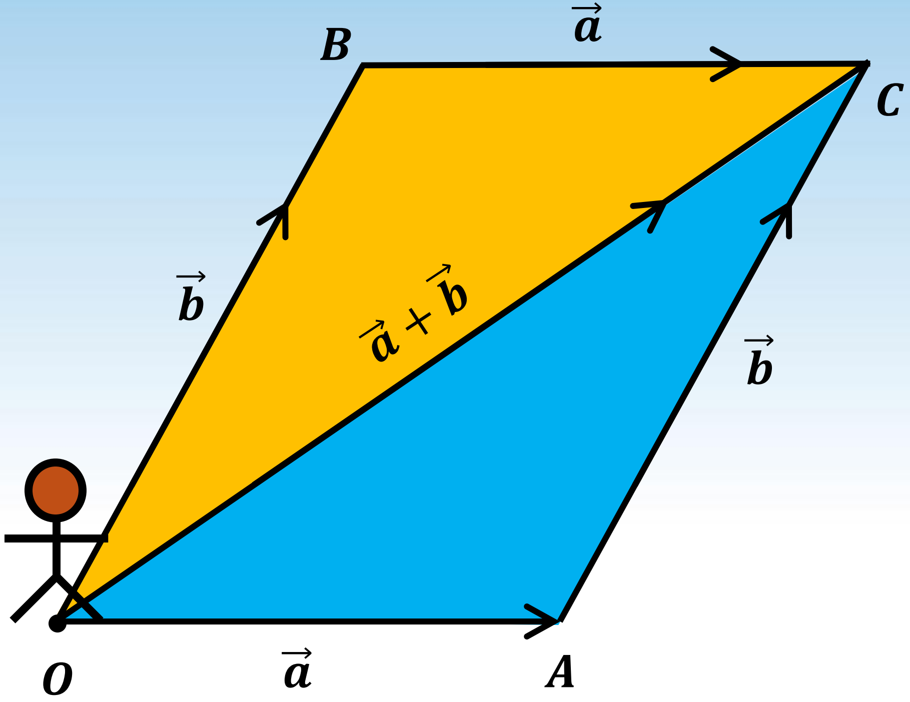


$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

$$\boxed{\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}}$$

Terminal Point

Initial Point



Parallelogram Law of Addition

$$\overrightarrow{OA}+\overrightarrow{AC}=\overrightarrow{OC} \Rightarrow \vec{a}+\vec{b}=\overrightarrow{OC}$$

$$\overrightarrow{OB}+\overrightarrow{BC}=\overrightarrow{OC} \Rightarrow \vec{b}+\vec{a}=\overrightarrow{OC}$$

$$\boxed{\vec{a}+\vec{b}=\vec{b}+\vec{a}}$$

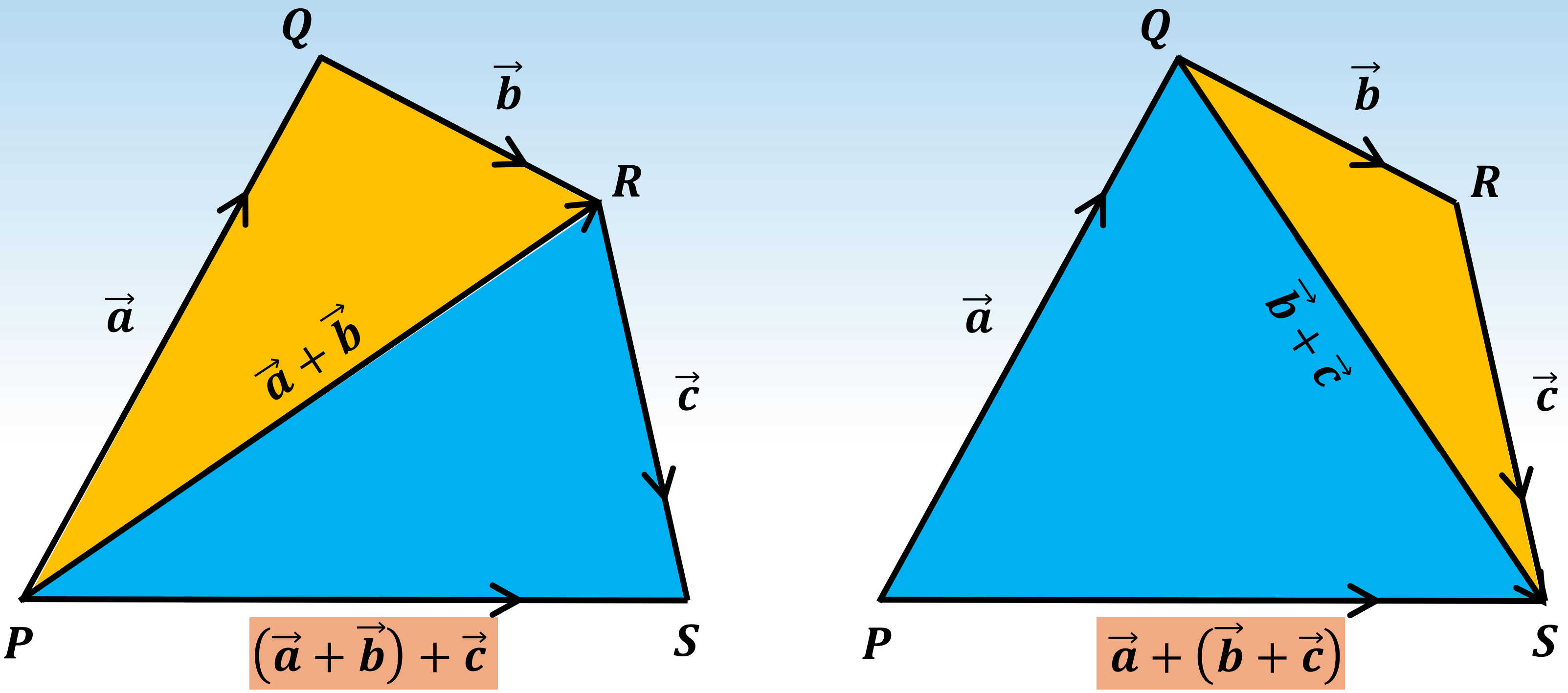
Commutative Property

$$\boxed{\vec{a}+\vec{0}=\vec{0}+\vec{a}=\vec{a}}$$

$\vec{0} \longrightarrow$ **Additive Identity**

$$(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$$

Associative Property



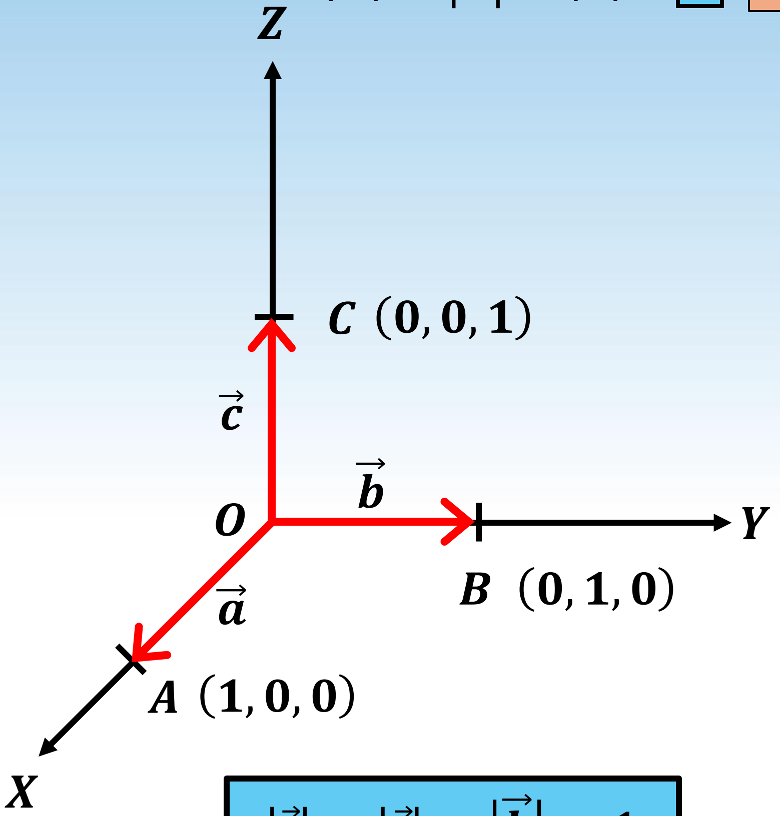
$|\vec{a}| = |\vec{b}| = |\vec{c}| = \boxed{1}$

Vector = Magnitude × Direction

$\vec{a} \longrightarrow |\vec{a}| = 1 \times \boxed{\hat{a}} \rightarrow \hat{\imath}$

$\vec{b} \longrightarrow |\vec{b}| = 1 \times \boxed{\hat{b}} \rightarrow \hat{\jmath}$

$\vec{c} \longrightarrow |\vec{c}| = 1 \times \boxed{\hat{c}} \rightarrow \hat{k}$



$|\vec{\imath}| = |\vec{\jmath}| = |\vec{k}| = 1$

$\vec{a} = |\vec{a}|\hat{a}$ | $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$ | $\vec{a} = \hat{\imath}$

$\vec{b} = |\vec{b}|\hat{b}$ | $\hat{b} = \dfrac{\vec{b}}{|\vec{b}|}$ | $\vec{b} = \hat{\jmath}$

$\vec{c} = |\vec{c}|\hat{c}$ | $\hat{c} = \dfrac{\vec{c}}{|\vec{c}|}$ | $\vec{c} = \hat{k}$

Unit Vectors along Axes

$\overrightarrow{OA} = x\hat{\imath}$

$\overrightarrow{OB} = y\hat{\jmath} = \overrightarrow{AD}$

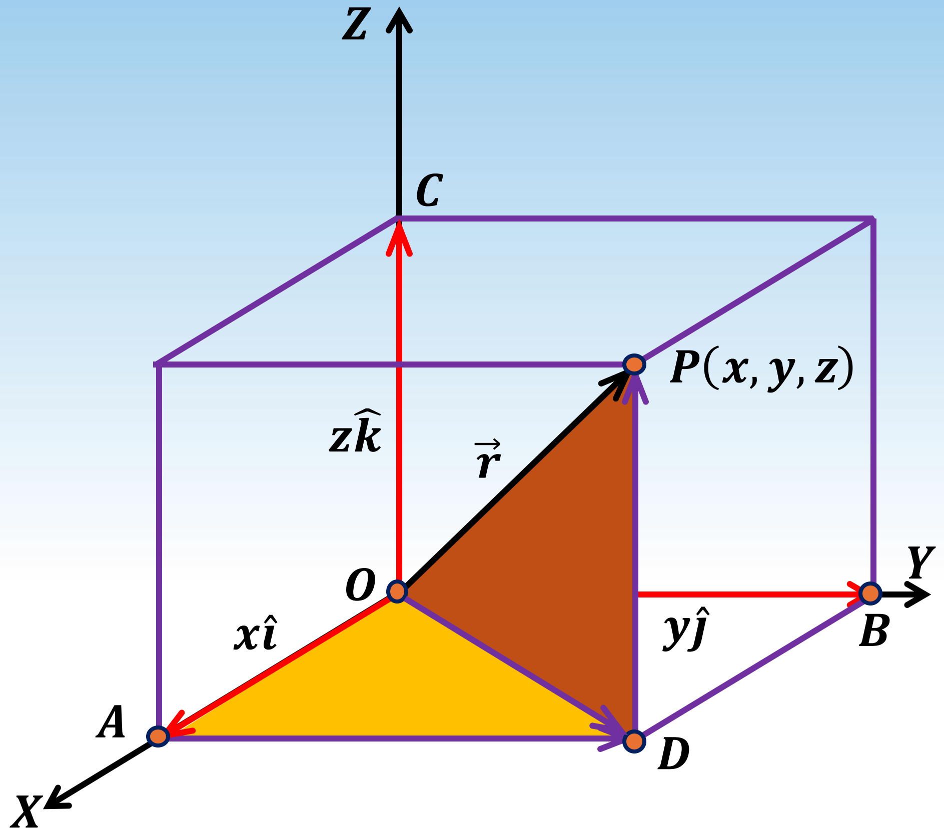
$\overrightarrow{OC} = z\hat{k} = \overrightarrow{DP}$

In ΔOAD

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = x\hat{\imath} + y\hat{\jmath}$

In ΔODP

$\overrightarrow{OP} = \overrightarrow{OD} + \overrightarrow{DP} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$



$$\boxed{\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}}$$

$$\boxed{|\vec{r}| = \sqrt{x^2 + y^2 + z^2}}$$

$$\boxed{\hat{r} = \frac{\vec{r}}{|\vec{r}|}}$$ → **Unit vector along $\vec{r}$**

Ex. Find the unit vector in the direction of the vector $3\hat{i} + \hat{j} + \hat{k}$.

Sol.

Ex. **Find the unit vector in the direction of the vector $3\hat{i}+\hat{j}+\hat{k}$.**

Sol.

$$\vec{r}=3\hat{\imath}+\hat{\jmath}+\hat{k}$$

$$|\vec{r}|=\sqrt{3^2+1^2+1^2}$$

$$\boxed{|\vec{r}|=\sqrt{11}}$$

$$\hat{r}=\frac{\vec{r}}{|\vec{r}|}=\frac{3\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{11}}$$

$$\boxed{\hat{r}=\frac{3}{\sqrt{11}}\hat{\imath}+\frac{1}{\sqrt{11}}\hat{\jmath}+\frac{1}{\sqrt{11}}\hat{k}}$$

$$\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k} \qquad \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

Sum : $\boxed{\vec{a} + \vec{b} = (a_1 + b_1)\hat{\imath} + (a_2 + b_2)\hat{\jmath} + (a_3 + b_3)\hat{k}}$

Difference : $\boxed{\vec{a} - \vec{b} = (a_1 - b_1)\hat{\imath} + (a_2 - b_2)\hat{\jmath} + (a_3 - b_3)\hat{k}}$

Equality : $\boxed{a_1 = b_1, a_2 = b_2 \;\&\; a_3 = b_3} \Rightarrow \boxed{\vec{a} = \vec{b}}$

Multiplication with scalars p and **q** :

$$\boxed{p\vec{a} = (pa_1)\hat{\imath} + (pa_2)\hat{\jmath} + (pa_3)\hat{k}} + \boxed{q\vec{a} = (qa_1)\hat{\imath} + (qa_2)\hat{\jmath} + (qa_3)\hat{k}}$$

$$p\vec{a} + q\vec{a} = (pa_1 + qa_1)\hat{\imath} + (pa_2 + qa_2)\hat{\jmath} + (pa_3 + qa_3)\hat{k}$$

$$p\vec{a} + q\vec{a} = (p+q)a_1\hat{\imath} + (p+q)a_2\hat{\jmath} + (p+q)a_3\hat{k}$$

Distributive Property

$$\boxed{p\vec{a} + q\vec{a} = (p+q)\vec{a}} \qquad \boxed{p(q\vec{a}) = (pq)\vec{a}} \qquad \boxed{p(\vec{a} + \vec{b}) = p\vec{a} + p\vec{b}}$$

Ex. Find the values of x and y so that the vectors $4\hat{i} + 5\hat{k}$ and $x\hat{i} + z\hat{k}$ are equal.

Sol.

$$\boxed{4\hat{\imath}} + \boxed{5\hat{k}} = \boxed{x\hat{\imath}} + \boxed{z\hat{k}}$$

$$\boxed{x = 4} \quad \boxed{z = 5}$$

Q. Find the sum and difference of the vectors $3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $-4\hat{\imath} + 8\hat{\jmath} + 3\hat{k}$.

Sol. $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ $\qquad \vec{b} = -4\hat{\imath} + 8\hat{\jmath} + 3\hat{k}$

$$\vec{a} + \vec{b} = \left(3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}\right) + \left(-4\hat{\imath} + 8\hat{\jmath} + 3\hat{k}\right)$$

$$\vec{a} + \vec{b} = (3 - 4)\hat{\imath} + (-2 + 8)\hat{\jmath} + (7 + 3)\hat{k}$$

$$\boxed{\vec{a} + \vec{b} = -\hat{\imath} + 6\hat{\jmath} + 10\hat{k}}$$

$$\vec{a} - \vec{b} = \left(3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}\right) - \left(-4\hat{\imath} + 8\hat{\jmath} + 3\hat{k}\right)$$

$$\vec{a} - \vec{b} = (3 + 4)\hat{\imath} + (-2 - 8)\hat{\jmath} + (7 - 3)\hat{k}$$

$$\boxed{\vec{a} - \vec{b} = 7\hat{\imath} - 10\hat{\jmath} + 4\hat{k}}$$

Collinear Vectors

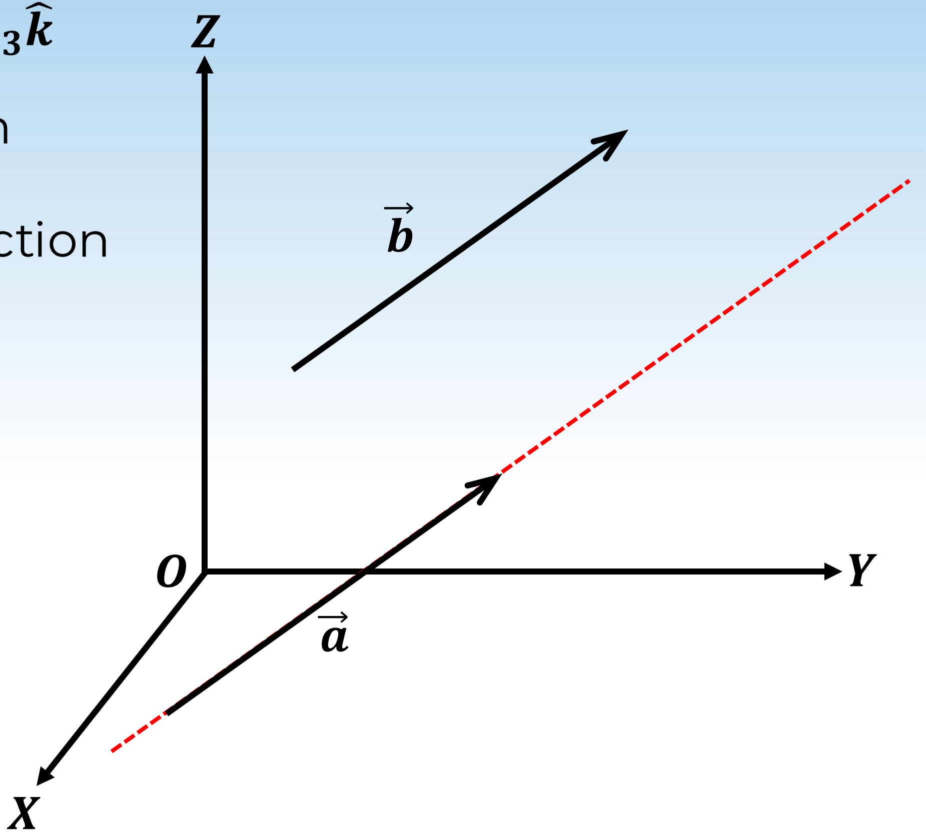
$\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ $\quad$ $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\boxed{\vec{b} = \lambda\vec{a}}$

$\lambda \rightarrow$ Scalar

$+$ Same Direction

$-$ Opposite Direction



Collinear Vectors

$\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ $\qquad$ $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$$\boxed{\vec{b} = \lambda\vec{a}}$$

$\lambda \rightarrow$ Scalar

$+$ Same Direction

$-$ Opposite Direction

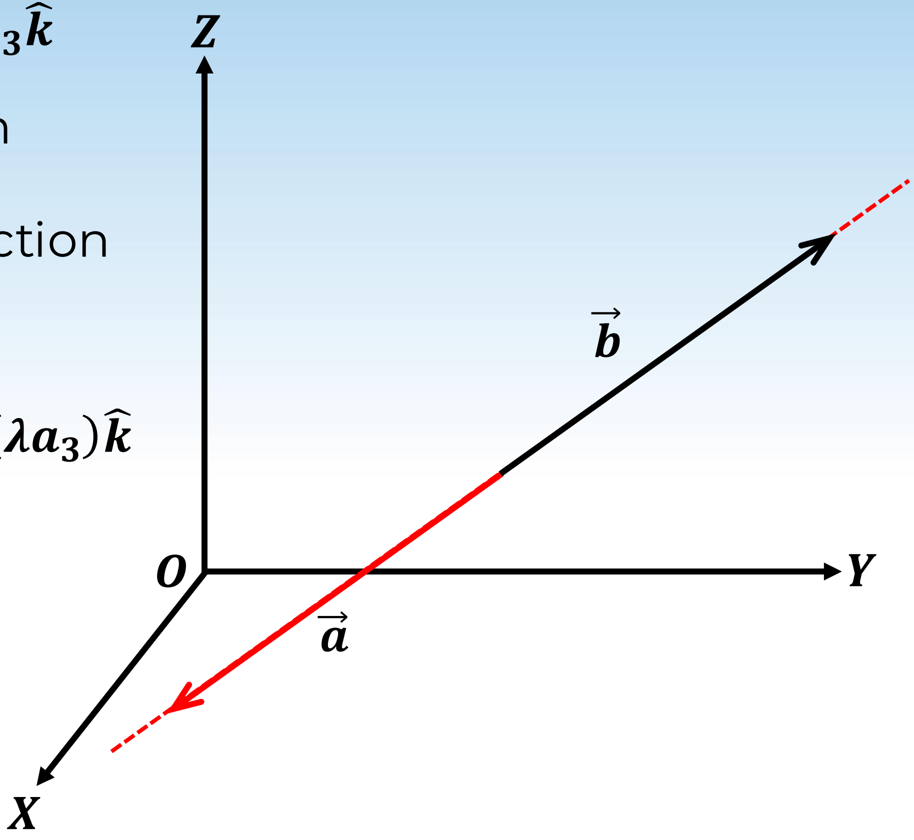
$$\Rightarrow \quad b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k} = \lambda\left(a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}\right)$$

$$\Rightarrow \quad b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k} = (\lambda a_1)\hat{\imath} + (\lambda a_2)\hat{\jmath} + (\lambda a_3)\hat{k}$$

$$\Rightarrow \quad b_1 = \lambda a_1 \qquad b_2 = \lambda a_2 \qquad b_3 = \lambda a_3$$

Condition for Collinearity

$$\boxed{\frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda}$$



Q. Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

Sol.

$\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$

$\vec{b} = \lambda\vec{a}$

$\vec{b} = -2(2\hat{i} - 3\hat{j} + 4\hat{k})$

$\vec{b} = -2\vec{a}$

$\vec{a}$ and $\vec{b}$ are collinear

$$\left.\begin{matrix}\alpha\\ \beta\\ \gamma\end{matrix}\right\}$$ **Direction Angles**

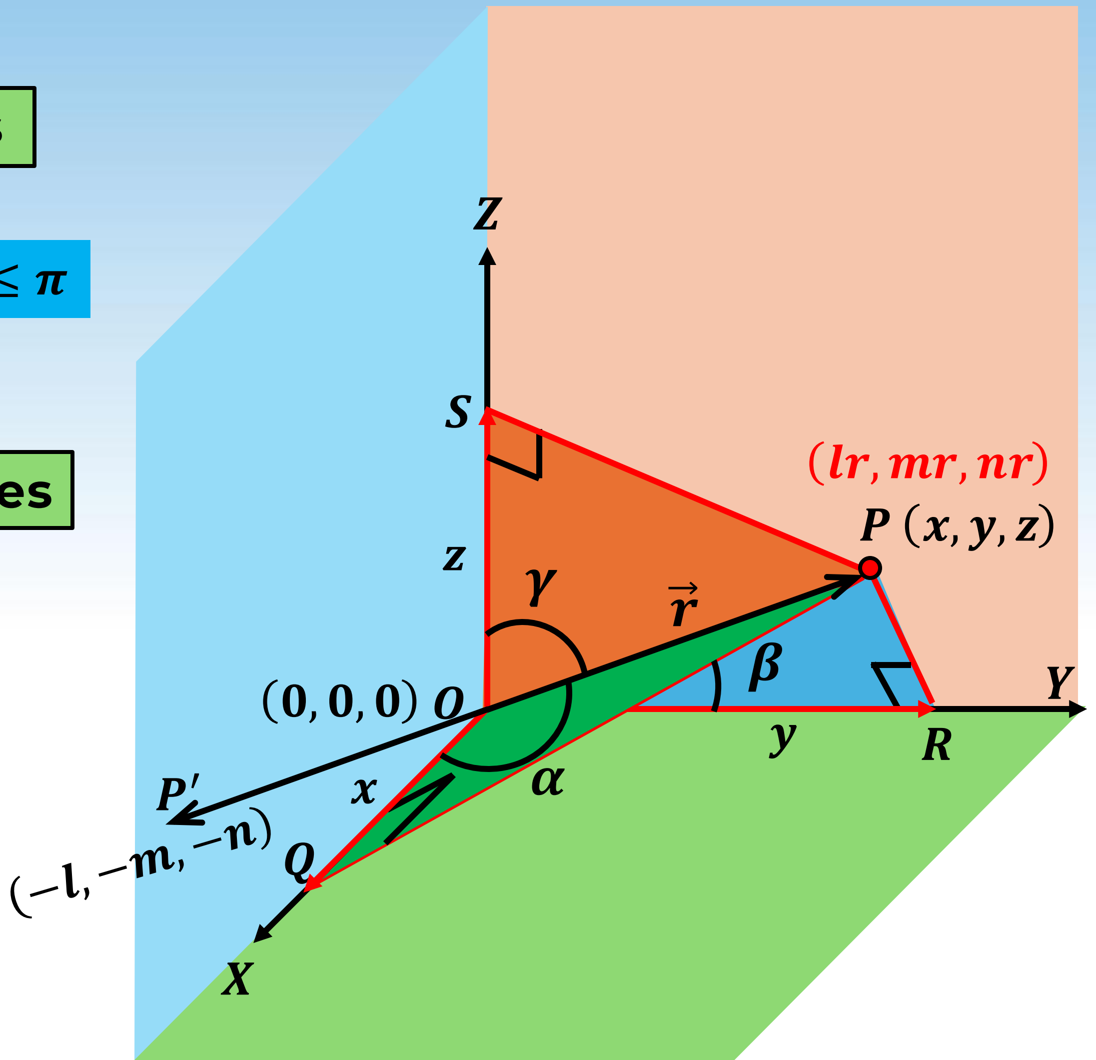
$0 \le \alpha \le \pi$	$0 \le \beta \le \pi$	$0 \le \gamma \le \pi$

$$\left.\begin{matrix} l = \cos\alpha \\ m = \cos\beta \\ n = \cos\gamma \end{matrix}\right\}$$ **Direction Cosines**

$$l = \cos\alpha = \frac{x}{r} \qquad \boxed{x = lr}$$

$$m = \cos\beta = \frac{y}{r} \qquad \boxed{y = mr}$$

$$n = \cos\gamma = \frac{z}{r} \qquad \boxed{z = nr}$$



Relation between l, m and n

$$l = \cos\alpha = \frac{x}{r}$$

$$m = \cos\beta = \frac{y}{r}$$

$$n = \cos\gamma = \frac{z}{r}$$

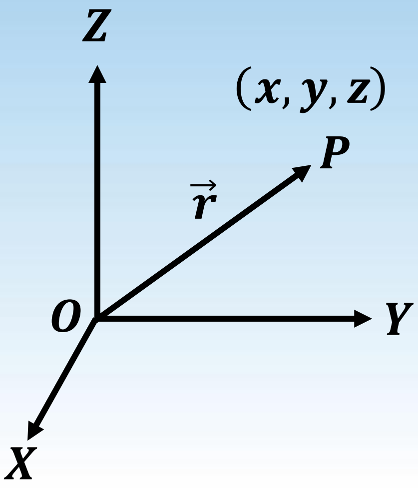
$$l^2 + m^2 + n^2 = \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 + \left(\frac{z}{r}\right)^2$$

$$l^2 + m^2 + n^2 = \frac{\boxed{x^2 + y^2 + z^2}}{r^2}$$

$$l^2 + m^2 + n^2 = \frac{r^2}{r^2}$$

$$\boxed{l^2 + m^2 + n^2 = 1}$$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$



$$\boxed{\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2 + z^2} = r}$$

$$\boxed{x^2 + y^2 + z^2 = r^2}$$

Ex. Find the **direction cosines** of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

Sol.

Ex. Find the **direction cosines** of the vector $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

Sol.

$x = 1 \qquad y = 2 \qquad z = 3$

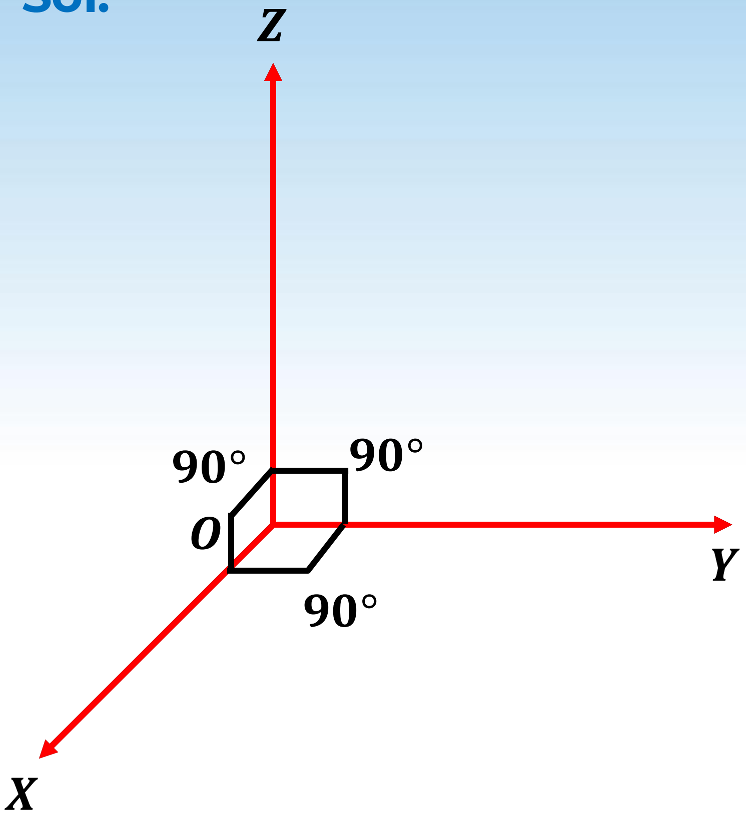
$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{1^2 + 2^2 + 3^2}$$

$$\boxed{r = \sqrt{14}}$$

$$\boxed{l = \frac{x}{r} = \frac{1}{\sqrt{14}}} \qquad \boxed{m = \frac{y}{r} = \frac{2}{\sqrt{14}}} \qquad \boxed{n = \frac{z}{r} = \frac{3}{\sqrt{14}}}$$

Q. Find the **direction cosines** of x – **axis**, y – **axis** and z – **axis**.

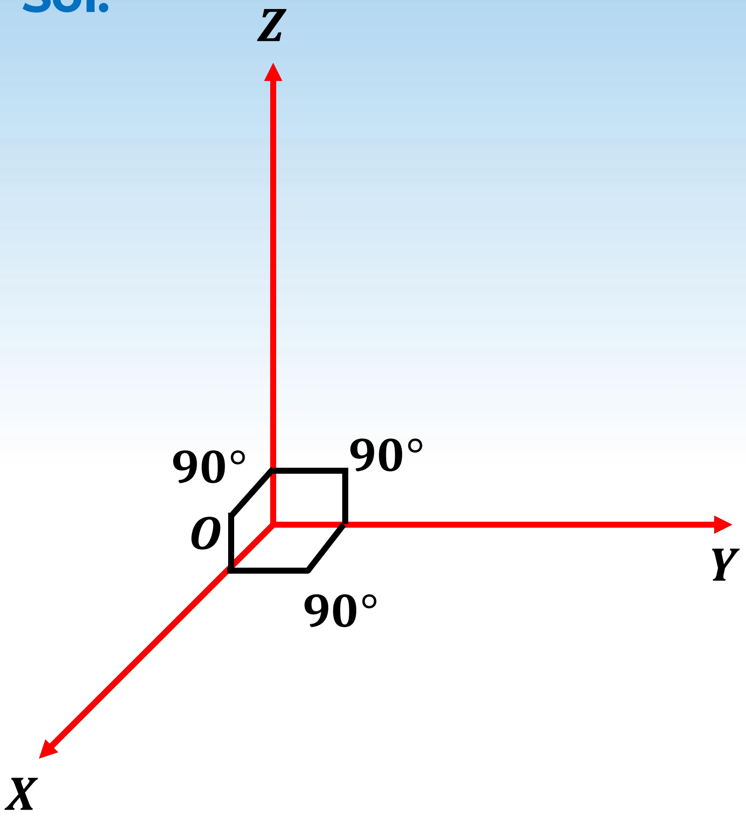
Sol.



$l = \cos\alpha$ $\quad m = \cos\beta$ $\quad n = \cos\gamma$

Q. Find the **direction cosines** of x – **axis**, y – **axis** and z – **axis**.

Sol.



$l = \cos\alpha \qquad m = \cos\beta \qquad n = \cos\gamma$

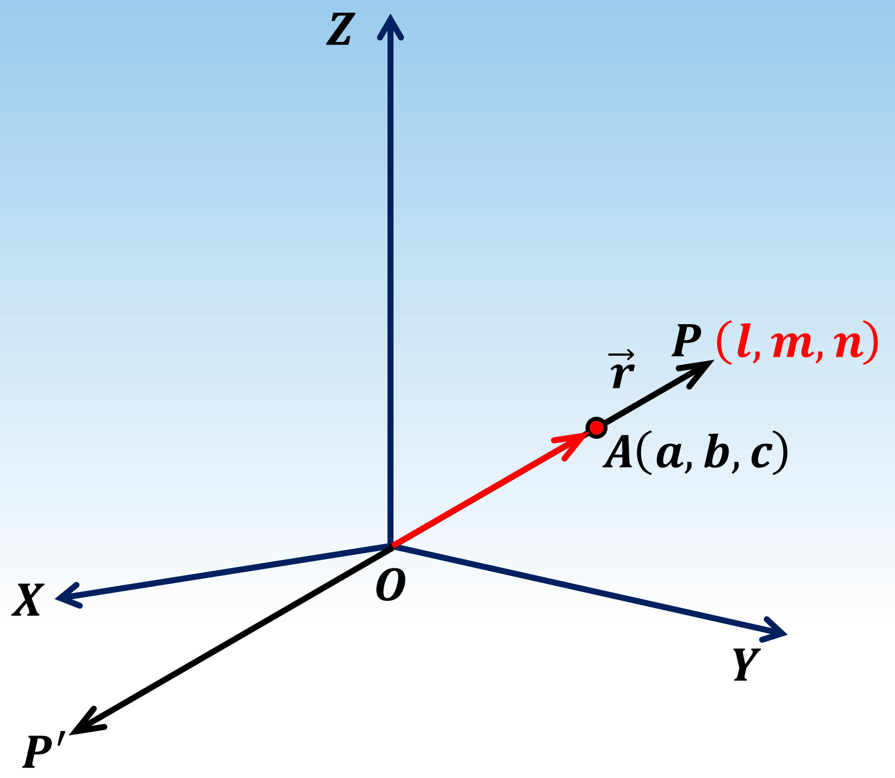
Axes	Direction Angles			Direction Cosines (l, m, n)
	α	β	γ	
X – Axis	$0°$	$90°$	$90°$	$(1, 0, 0)$
Y – Axis	$90°$	$0°$	$90°$	$(0, 1, 0)$
Z – Axis	$90°$	$90°$	$0°$	$(0, 0, 1)$

$$l = \lambda a \qquad m = \lambda b \qquad n = \lambda c$$

$$l = \pm \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \pm \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \pm \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$



$$\lambda = \pm \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

Ex. Find the **direction cosines** of the vector whose **direction ratios** are $2, -1, -2$.

Sol.

Ex. Find the direction cosines of the vector whose direction ratios are $2, -1, -2$.

Sol. $a = 2 \quad b = -1 \quad c = -2$

$$\lambda = \pm\frac{1}{\sqrt{a^2+b^2+c^2}} = \pm\frac{1}{\sqrt{2^2+(-1)^2+(-2)^2}} = \pm\frac{1}{\sqrt{4+1+4}} = \pm\frac{1}{\sqrt{9}}$$

$$\boxed{\lambda = \pm\frac{1}{3}}$$

$l = \lambda a$

$l = \left(\pm\frac{1}{3}\right)\times 2$

$$\boxed{l = \pm\frac{2}{3}}$$

$m = \lambda b$

$m = \left(\pm\frac{1}{3}\right)\times(-1)$

$$\boxed{m = \mp\frac{1}{3}}$$

$n = \lambda c$

$n = \left(\pm\frac{1}{3}\right)\times(-2)$

$$\boxed{n = \mp\frac{2}{3}}$$

l	m	n
$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$
$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$

Vector joining two points

$\overrightarrow{OP_1} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ $\quad$ $\overrightarrow{OP_2} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$

$\overrightarrow{OP_1} + \overrightarrow{P_1P_2} = \overrightarrow{OP_2} \longrightarrow$ **Triangle Law**

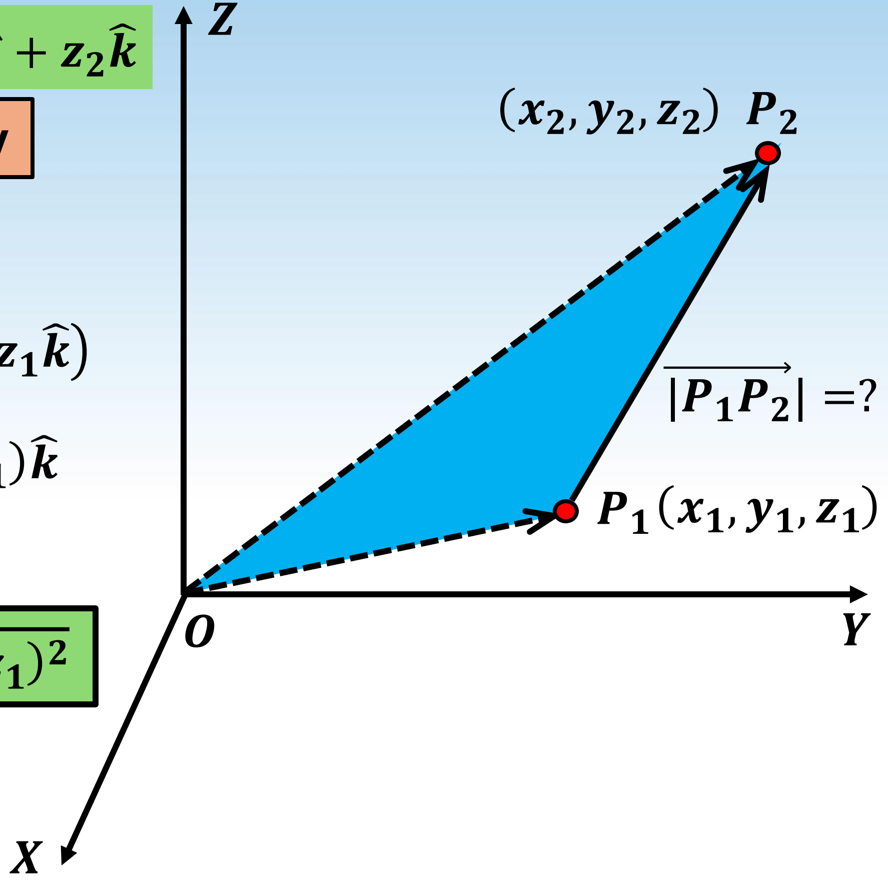
$$\boxed{\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1}}$$

$$\overrightarrow{P_1P_2} = \left(x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}\right) - \left(x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}\right)$$

$$\overrightarrow{P_1P_2} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$$

Distance Formula

$$\left|\overrightarrow{P_1P_2}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



Section Formula

Internal Division

$$\frac{\overrightarrow{PR}}{\overrightarrow{RQ}} = \frac{m}{n}$$

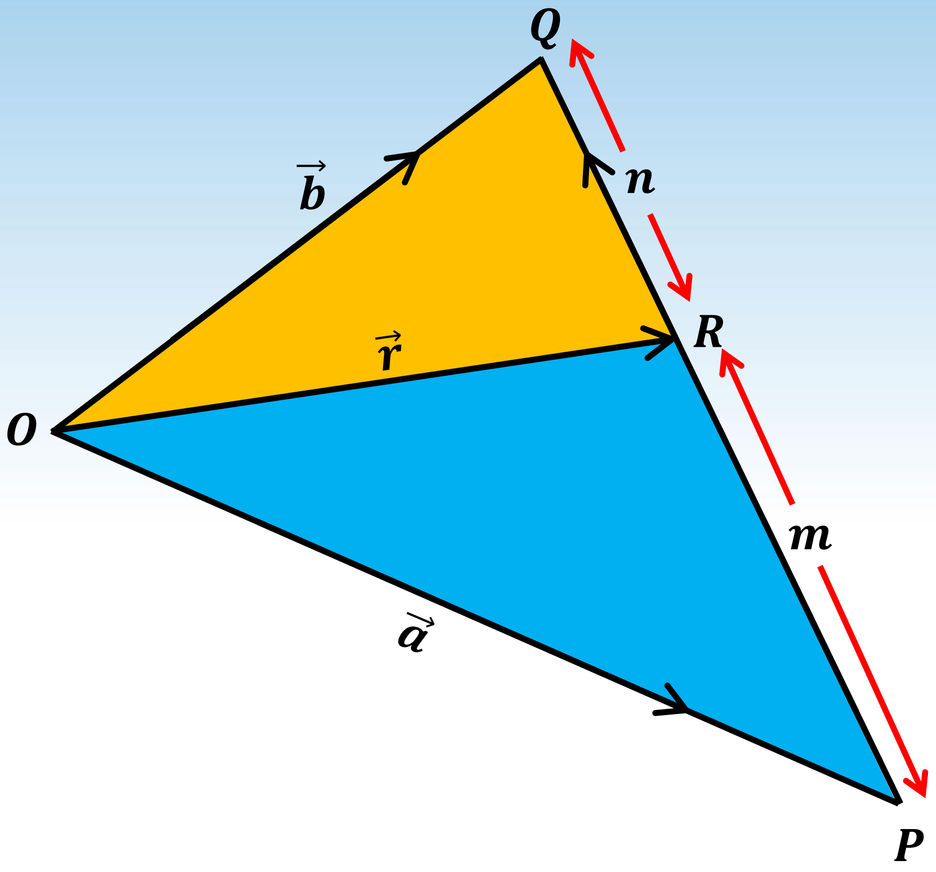
$$m\overrightarrow{RQ} = n\overrightarrow{PR}$$

$$\overrightarrow{RQ} = \overrightarrow{OQ} - \overrightarrow{OR} = \vec{b} - \vec{r}$$

$$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP} = \vec{r} - \vec{a}$$

$$m(\vec{b} - \vec{r}) = n(\vec{r} - \vec{a})$$

$$\boxed{\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}}$$



Section Formula

$$\frac{\overrightarrow{PR}}{\overrightarrow{RQ}} = \frac{m}{n} = 1$$

$$\overrightarrow{RQ} = \overrightarrow{PR}$$

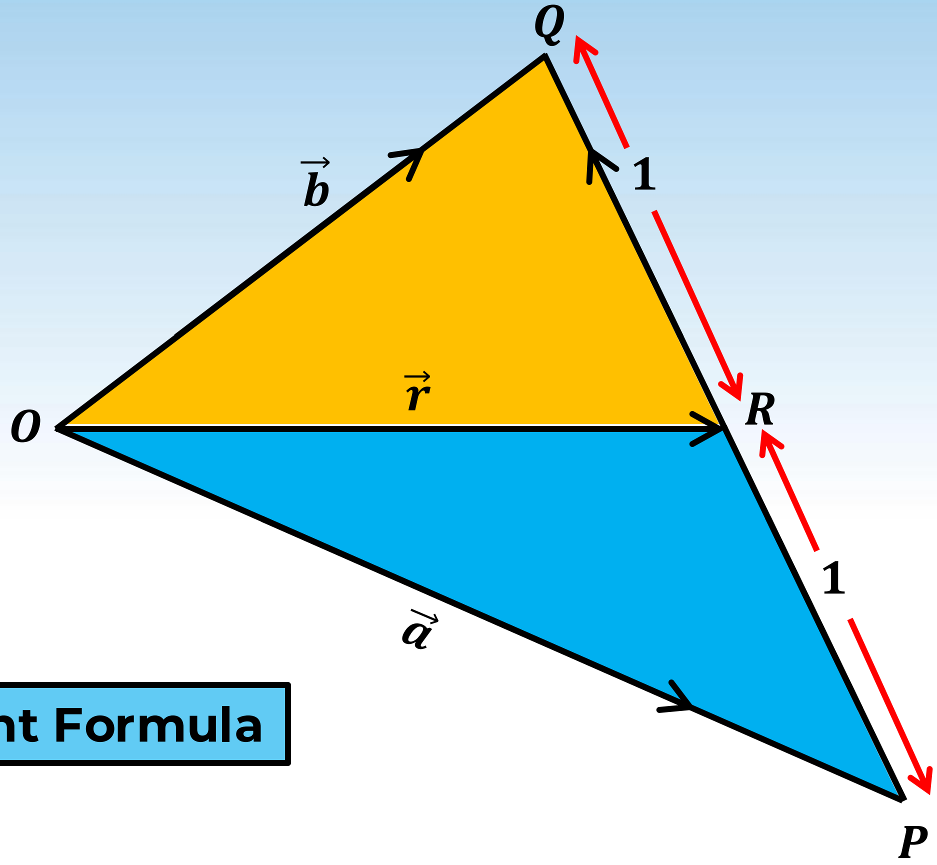
$$\overrightarrow{RQ} = \overrightarrow{OQ} - \overrightarrow{OR} = \vec{b} - \vec{r}$$

$$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP} = \vec{r} - \vec{a}$$

$$\vec{b} - \vec{r} = \vec{r} - \vec{a}$$

$$\boxed{\vec{r} = \frac{\vec{b} + \vec{a}}{2}}$$

Mid – Point Formula



Section Formula

External Division

$$\frac{\overrightarrow{PR}}{\overrightarrow{QR}} = \frac{m}{n}$$

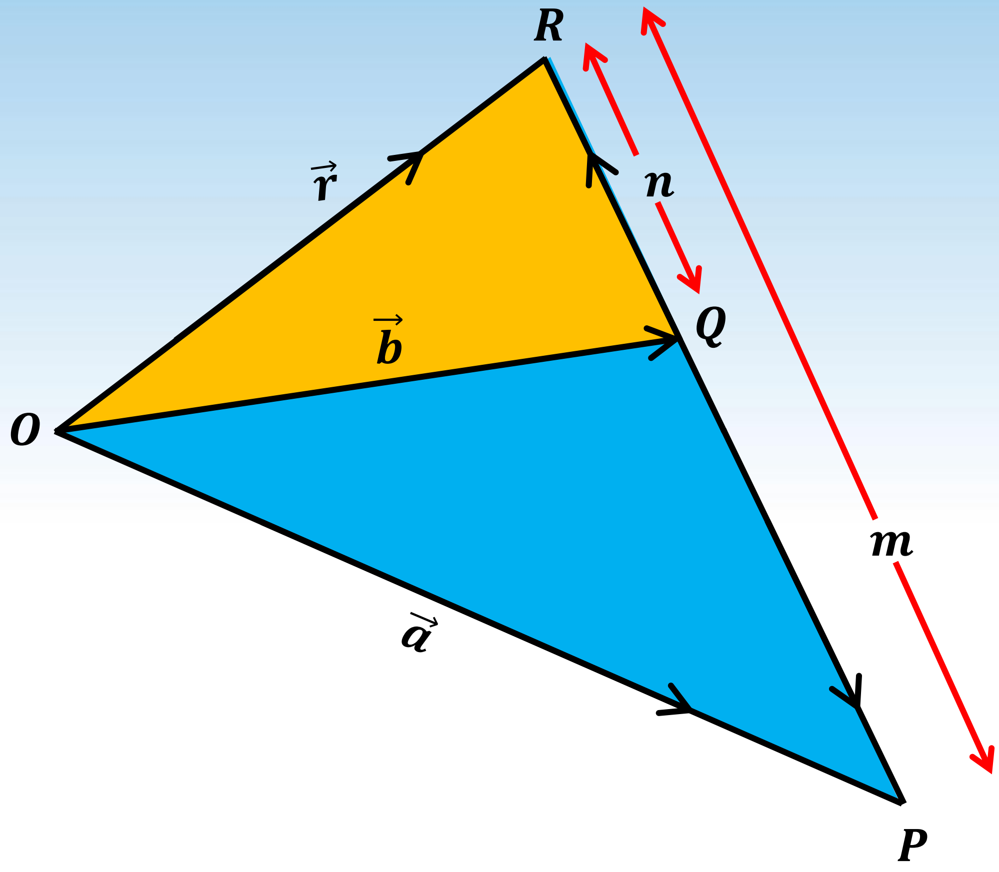
$$m\overrightarrow{QR} = n\overrightarrow{PR}$$

$$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ} = \vec{r} - \vec{b}$$

$$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP} = \vec{r} - \vec{a}$$

$$m(\vec{r} - \vec{b}) = n(\vec{r} - \vec{a})$$

$$\boxed{\vec{r} = \frac{m\vec{b} - n\vec{a}}{m - n}}$$



Q. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively, in the ratio $2:1$

(i) internally (ii) externally.

Sol.

(i)

(ii)

Q. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $-\hat{\imath} + \hat{\jmath} + \hat{k}$ respectively, in the ratio $2:1$

(i) internally (ii) externally.

Sol. $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ $\quad \vec{b} = -\hat{\imath} + \hat{\jmath} + \hat{k}$ $\quad \dfrac{m}{n} = \dfrac{2}{1}$

(i)

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m + n}$$

$$\vec{r} = \frac{2(-\hat{\imath} + \hat{\jmath} + \hat{k}) + (\hat{\imath} + 2\hat{\jmath} - \hat{k})}{2 + 1}$$

$$\boxed{\vec{r} = -\frac{1}{3}\hat{\imath} + \frac{4}{3}\hat{\jmath} + \frac{1}{3}\hat{k}}$$

(ii)

$$\vec{r} = \frac{m\vec{b} - n\vec{a}}{m - n}$$

$$\vec{r} = \frac{2(-\hat{\imath} + \hat{\jmath} + \hat{k}) - (\hat{\imath} + 2\hat{\jmath} - \hat{k})}{2 - 1}$$

$$\boxed{\vec{r} = -3\hat{\imath} + 3\hat{k}}$$

Summary

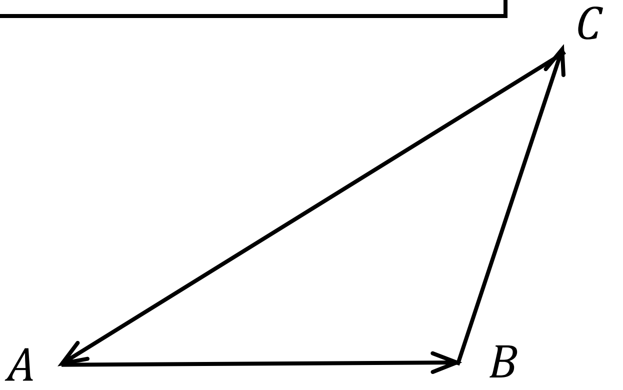
Direction Cosines → $l = \cos\alpha$ | $m = \cos\beta$ | $n = \cos\gamma$

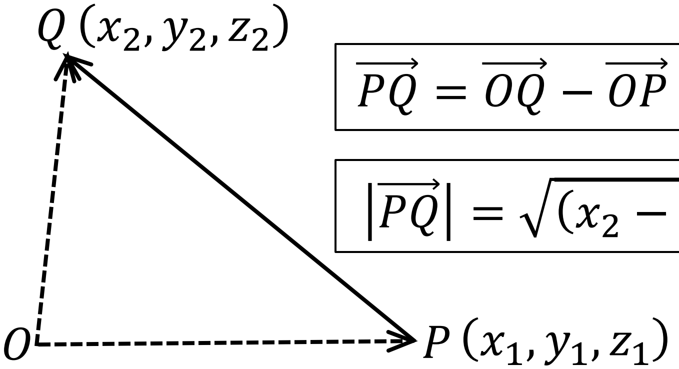
$l^2 + m^2 + n^2 = 1$

$\cos\alpha^2 + \cos\beta^2 + \cos\gamma^2 = 1$

Commutative Property : $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

Associative Property : $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$





$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

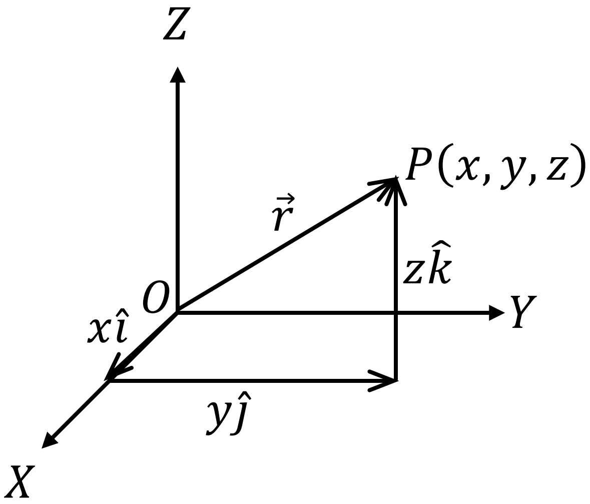
Distance Formula

$$\left|\overrightarrow{PQ}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Triangle Rule

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$

Summary



$$\overrightarrow{OP} = \vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

$$|\hat{\imath}| = |\hat{\jmath}| = |\hat{k}| = 1$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\vec{a} \pm \vec{b} = (a_1 \pm b_1)\hat{\imath} + (a_2 \pm b_2)\hat{\jmath} + (a_3 \pm b_3)\hat{k}$$

$$\vec{a} = \vec{b} \Longrightarrow a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$a_1 = b_1 \qquad a_2 = b_2 \qquad a_3 = b_3$$

$$\lambda\vec{a} = \lambda a_1\hat{\imath} + \lambda a_2\hat{\jmath} + \lambda a_3\hat{k} \longrightarrow \lambda \rightarrow \text{Scalar}$$

Collinear Vectors

$$b = \lambda a$$

$$\frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda$$

Section Formula

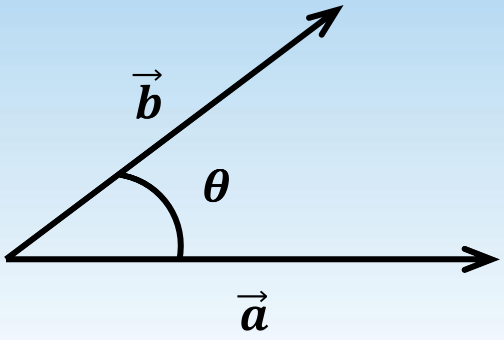
$$\vec{r} = \frac{m\vec{b} \pm n\vec{a}}{m \pm n}$$

Scalar / Dot Product of Two Vectors

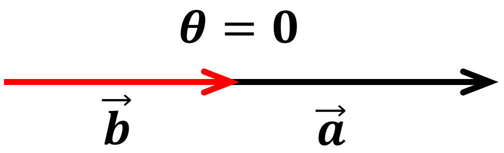
$\boxed{\begin{matrix}\vec{a}\\ \vec{b}\end{matrix}}$ Non Zero Vectors

$$\boxed{\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta} \rightarrow \text{Scalar}$$

$$0 \le \theta \le \pi$$

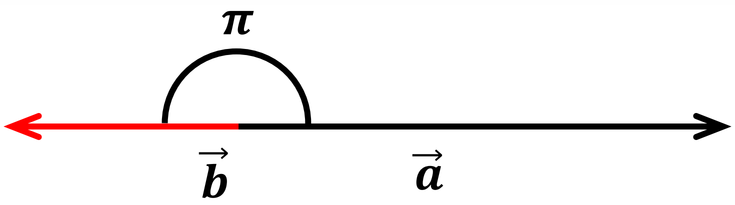


$$\boxed{\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}} \qquad \boxed{\theta = \cos^{-1}\left[\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right]}$$



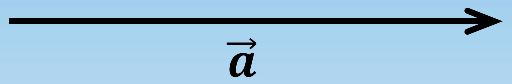
$$\boxed{\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos 0 = |\vec{a}||\vec{b}|}$$

$\vec{a}\cdot\vec{b}$ is Maximum



$$\boxed{\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\pi = -|\vec{a}||\vec{b}|}$$

$\vec{a}\cdot\vec{b}$ is Minimum



$$\vec{a}\cdot\vec{a} = |\vec{a}||\vec{a}|\cos 0 = |\vec{a}||\vec{a}|$$

$$\boxed{\vec{a}\cdot\vec{a} = |\vec{a}|^2}$$

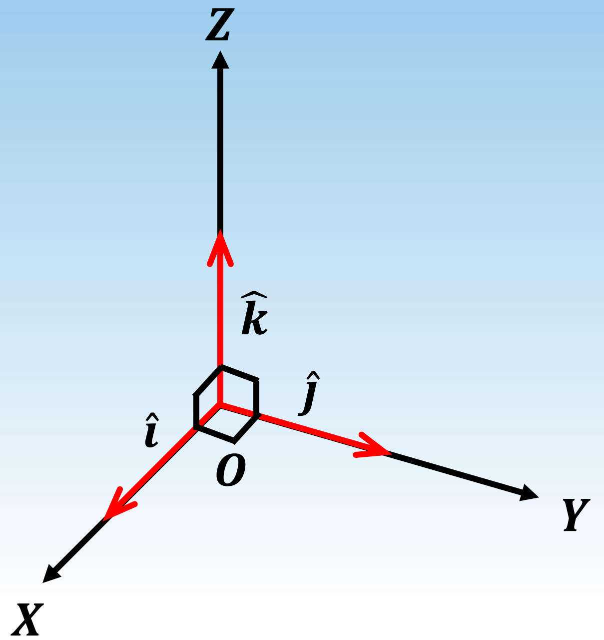
$$|\hat{\imath}| = |\hat{\jmath}| = |\hat{k}| = 1$$

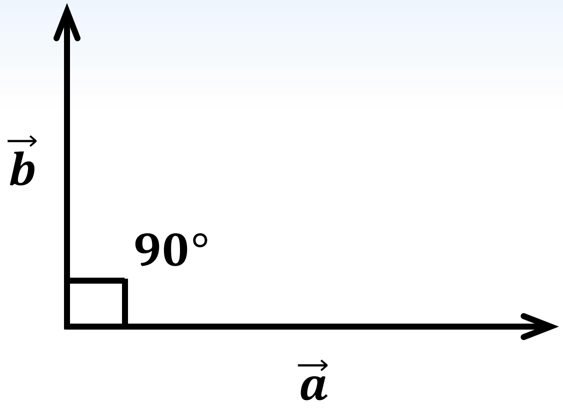
$$\hat{\imath}\cdot\hat{\imath} = 1\times 1\cos 0 = 1$$

$$\hat{\jmath}\cdot\hat{\jmath} = 1\times 1\cos 0 = 1$$

$$\hat{k}\cdot\hat{k} = 1\times 1\cos 0 = 1$$

$$\boxed{\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1}$$





$$\hat{\imath}\cdot\hat{\jmath} = 1\times 1\cos 90^\circ = 0$$

$$\hat{\jmath}\cdot\hat{k} = 1\times 1\cos 90^\circ = 0$$

$$\hat{k}\cdot\hat{\imath} = 1\times 1\cos 90^\circ = 0$$

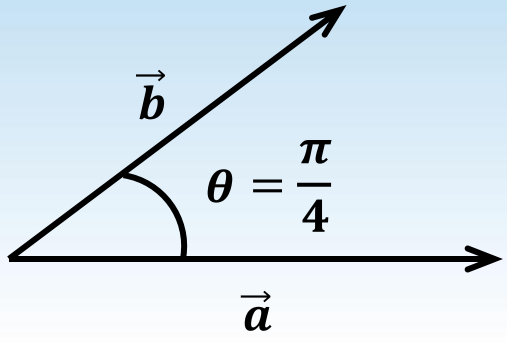
$$\boxed{\hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{k} = \hat{k}\cdot\hat{\imath} = 0}$$

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos 90^\circ = 0$$

Ex. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2, respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$.

Sol.

$|\vec{a}| = \sqrt{3} \qquad |\vec{b}| = 2 \qquad \vec{a} \cdot \vec{b} = \sqrt{6}$



$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\cos\theta = \frac{\sqrt{6}}{\sqrt{3} \times 2} = \frac{1}{\sqrt{2}}$$

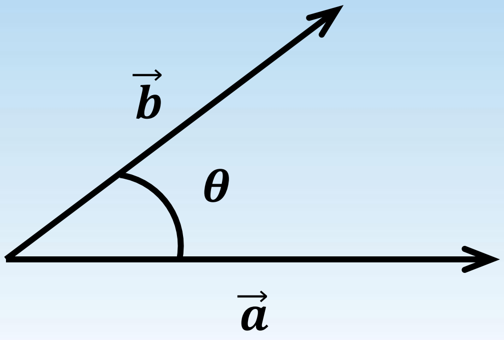
$$\boxed{\theta = \frac{\pi}{4}}$$

Properties of Scalar Products

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\vec{b}\cdot\vec{a} = |\vec{b}||\vec{a}|\cos\theta$$

Scalars



$$\vec{b}\cdot\vec{a} = \vec{a}\cdot\vec{b}$$

Commutative Property

$$\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$$

Distributive Property

$$(\lambda\vec{a})\cdot\vec{b} = \lambda(\vec{a}\cdot\vec{b}) = \vec{a}\cdot(\lambda\vec{b})$$

Ex. Find the scalar product of the vectors $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{b} = -\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$.

Sol.

Ex. Find the scalar product of the vectors $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{b} = -\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$.

Sol.

$$\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k} \qquad \vec{b} = -\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$$

$$\vec{a} \cdot \vec{b} = \left(2\hat{\imath} + 3\hat{\jmath} - \hat{k}\right) \cdot \left(-\hat{\imath} + 5\hat{\jmath} + 3\hat{k}\right)$$

$$\vec{a} \cdot \vec{b} = (2\hat{\imath}) \cdot (-\hat{\imath}) + (3\hat{\jmath}) \cdot (5\hat{\jmath}) + \left(-\hat{k}\right) \cdot \left(3\hat{k}\right)$$

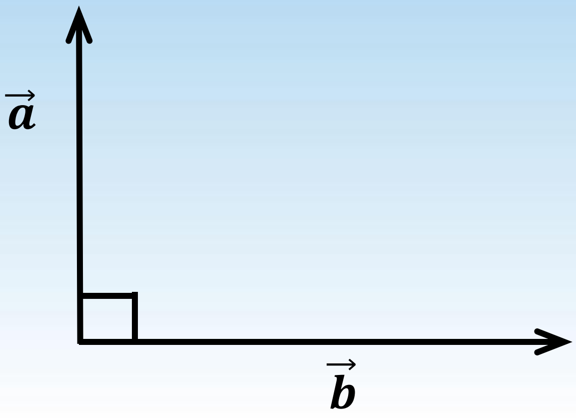
$$\vec{a} \cdot \vec{b} = -2 + 15 - 3$$

$$\boxed{\vec{a} \cdot \vec{b} = 10}$$

Q. Find the value of y if the vectors $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + y\hat{\jmath} + 4\hat{k}$ are perpendicular to each other.

Sol.

$$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k} \qquad \vec{b} = \hat{\imath} + y\hat{\jmath} + 4\hat{k}$$



$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos 90^\circ = 0$$

$$[2\hat{\imath} - 3\hat{\jmath} + \hat{k}] \cdot [\hat{\imath} + y\hat{\jmath} + 4\hat{k}] = 0$$

$$2[\hat{\imath} \cdot \hat{\imath}] - 3y[\hat{\jmath} \cdot \hat{\jmath}] + 4[\hat{k} \cdot \hat{k}] = 0$$

$$2 - 3y + 4 = 0$$

$$\boxed{y = 2}$$

Vector / Cross Product of Two Vectors

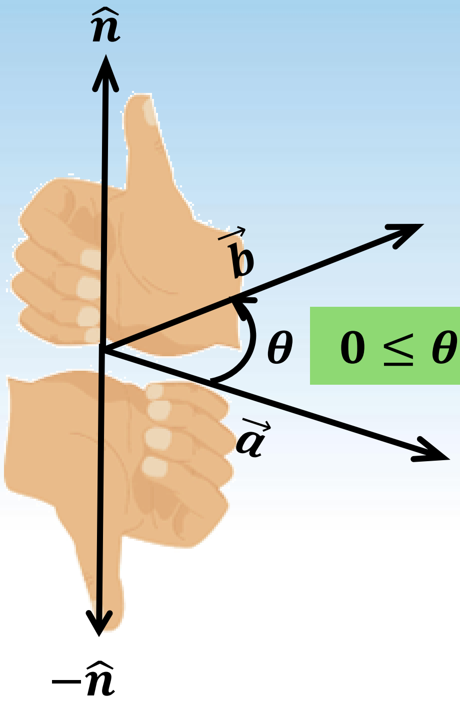
$\vec{a}$, $\vec{b}$ Non Zero Vectors

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta\, \hat{n}$$

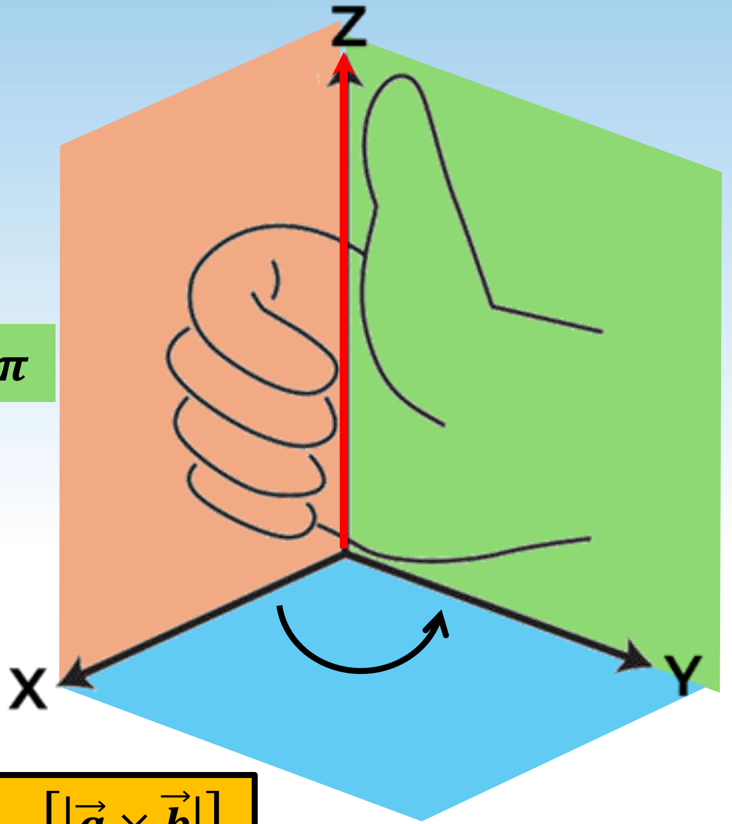
$$\vec{b} \times \vec{a} = |\vec{b}||\vec{a}| \sin\theta\, (-\hat{n})$$

$$\vec{a} \times \vec{b} = -[\vec{b} \times \vec{a}]$$

$$|\hat{n}| = |-\hat{n}| = 1$$



$$0 \le \theta \le \pi$$



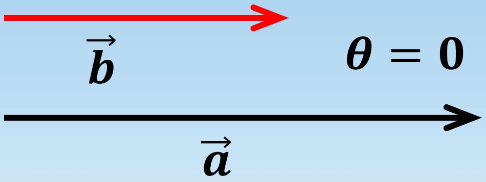
$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$

$$|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{a}|$$

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$$

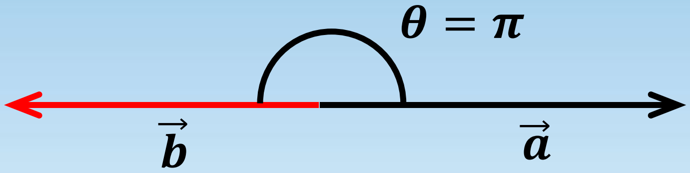
$$\theta = \sin^{-1}\left[\frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}\right]$$

Observations



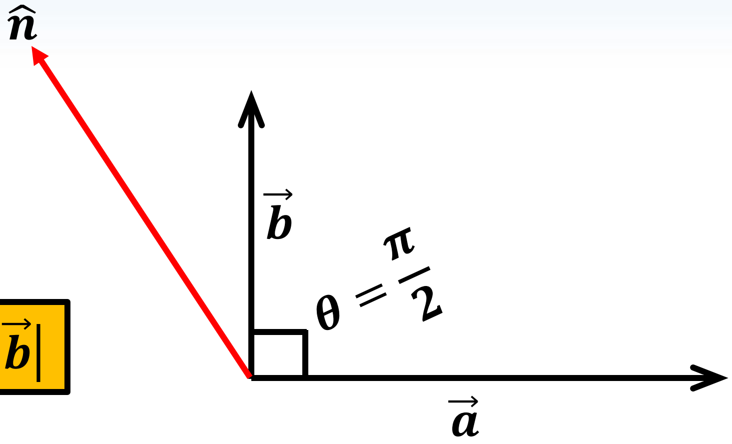
$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin 0 \, \hat{n}$$

$$\boxed{\vec{a} \times \vec{b} = 0} \quad \boxed{|\vec{a} \times \vec{b}| = 0}$$



$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin \pi \, \hat{n}$$

$$\boxed{\vec{a} \times \vec{b} = 0} \quad \boxed{|\vec{a} \times \vec{b}| = 0}$$



$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin \left(\frac{\pi}{2}\right) \hat{n}$$

$$\boxed{\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\hat{n}} \quad \boxed{|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|}$$

Ex. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Sol.

Ex. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is

(A) $\frac{\pi}{6}$ (B) $\boxed{\frac{\pi}{4}}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Sol.

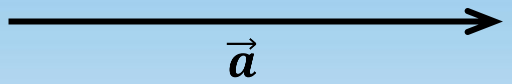
$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta\, \hat{n}$$

$$|\vec{a} \times \vec{b}| = 1$$

$$\Rightarrow |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$

$$\Rightarrow 1 = 3 \times \frac{\sqrt{2}}{3} \sin\theta$$

$$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4} \Rightarrow \boxed{\theta = \frac{\pi}{4}}$$



$$\vec{a} \times \vec{a} = |\vec{a}||\vec{a}|(\sin 0°)\hat{n} = 0$$

$$\boxed{\vec{a} \times \vec{a} = 0} \quad \boxed{|\vec{a} \times \vec{a}| = 0}$$

$$|\hat{\imath}| = |\hat{\jmath}| = |\hat{k}| = 1$$

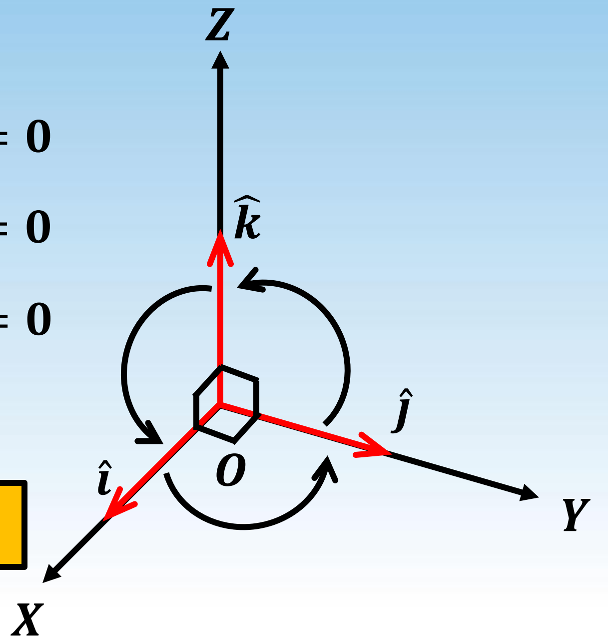
$$\hat{\imath} \times \hat{\imath} = 1 \times 1(\sin 0°)\hat{n}_1 = 0$$

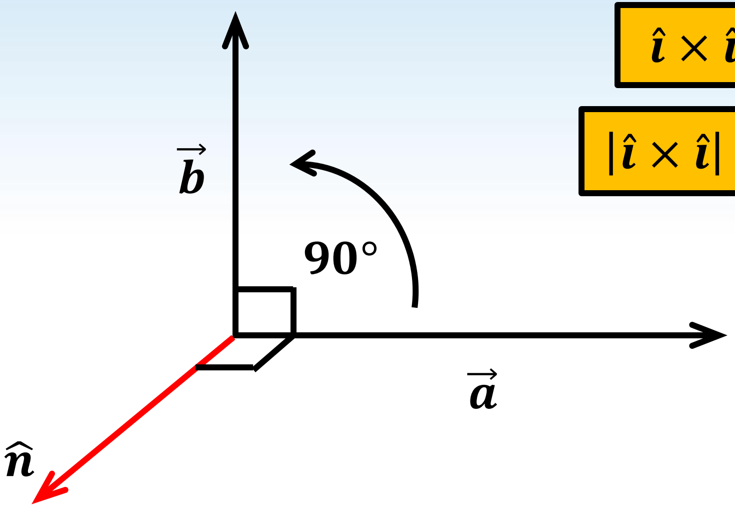
$$\hat{\jmath} \times \hat{\jmath} = 1 \times 1(\sin 0°)\hat{n}_2 = 0$$

$$\hat{k} \times \hat{k} = 1 \times 1(\sin 0°)\hat{n}_3 = 0$$

$$\boxed{\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0}$$

$$\boxed{|\hat{\imath} \times \hat{\imath}| = |\hat{\jmath} \times \hat{\jmath}| = |\hat{k} \times \hat{k}| = 0}$$



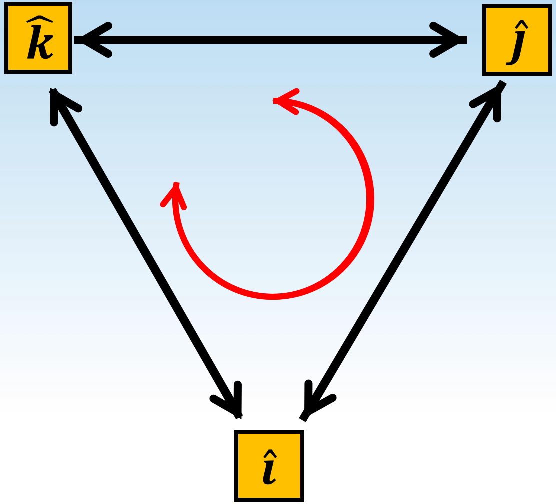


$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|(\sin 90°)\hat{n} = |\vec{a}||\vec{b}|\hat{n}$$

$$\hat{\imath} \times \hat{\jmath} = 1 \times 1 \sin 90° \boxed{\hat{n}_1} = \hat{k} \qquad \boxed{\hat{\imath} \times \hat{\jmath} = \hat{k}}$$

$$\hat{\jmath} \times \hat{k} = 1 \times 1 \sin 90° \boxed{\hat{n}_2} = \hat{\imath} \qquad \boxed{\hat{\jmath} \times \hat{k} = \hat{\imath}}$$

$$\hat{k} \times \hat{\imath} = 1 \times 1 \sin 90° \boxed{\hat{n}_3} = \hat{\jmath} \qquad \boxed{\hat{k} \times \hat{\imath} = \hat{\jmath}}$$



$$\hat{\jmath} \times \hat{\imath} = -\hat{k}$$

$$\hat{k} \times \hat{\jmath} = -\hat{\imath}$$

$$\hat{\imath} \times \hat{k} = -\hat{\jmath}$$

$$\hat{\imath} \times \hat{\jmath} = \hat{k}$$

$$\hat{\jmath} \times \hat{k} = \hat{\imath}$$

$$\hat{\imath} = \hat{\jmath}$$

Distributive Property

$$\vec{a} \times \left(\vec{b} + \vec{c}\right) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$\lambda \rightarrow$ **Scalar**

$$\lambda\left[\vec{a} \times \vec{b}\right] = \left[\lambda\vec{a}\right] \times \vec{b} = \vec{a} \times \left[\lambda\vec{b}\right]$$

Q. Find λ and μ if $\boxed{\left[2\hat{i} + 6\hat{j} + 27\hat{k}\right] \times \left[\hat{i} + \lambda\hat{j} + \mu\hat{k}\right] = 0}$.

Sol.

Q. Find λ and μ if $\boxed{[2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}] \times [\hat{\imath} + \lambda\hat{\jmath} + \mu\hat{k}] = 0}$.

Sol.

$$[2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}] \times [\hat{\imath} + \lambda\hat{\jmath} + \mu\hat{k}] = \boxed{\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix}}$$

$$0\hat{\imath} + 0\hat{\jmath} + 0\hat{k} = \begin{vmatrix} 6 & 27 \\ \lambda & \mu \end{vmatrix}\hat{\imath} + \begin{vmatrix} 27 & 2 \\ \mu & 1 \end{vmatrix}\hat{\jmath} + \begin{vmatrix} 2 & 6 \\ 1 & \lambda \end{vmatrix}\hat{k}$$

$$\boxed{0\hat{\imath}} + \boxed{0\hat{\jmath}} + \boxed{0\hat{k}} = \boxed{[6\mu - 27\lambda]\hat{\imath}} + \boxed{[27 - 2\mu]\hat{\jmath}} + \boxed{[2\lambda_2 - 6]\hat{k}}$$

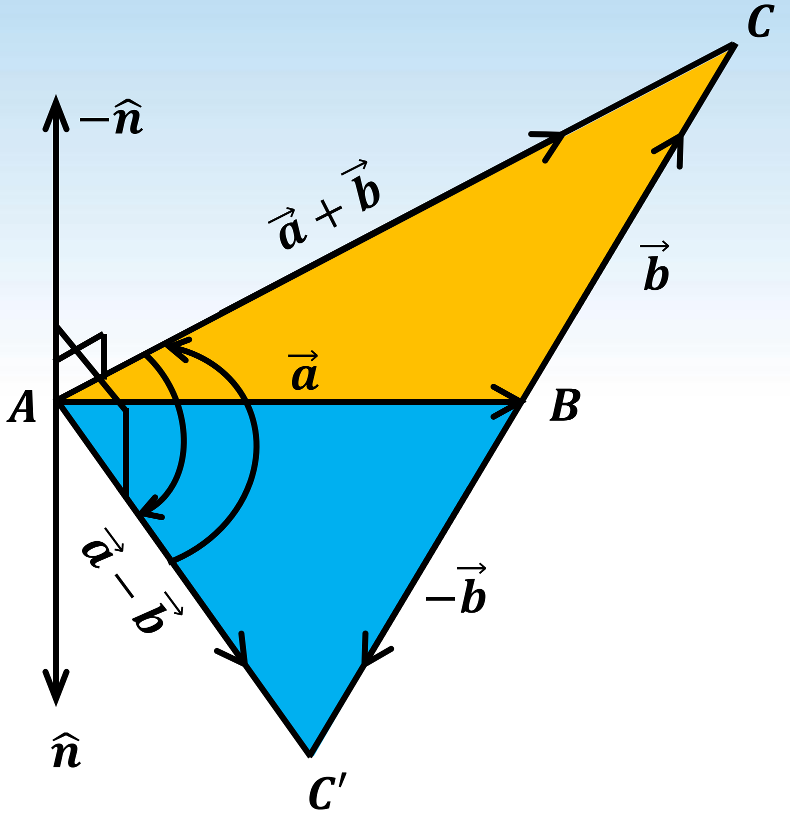
$$\boxed{0 = 6\mu - 27\lambda} \qquad \boxed{0 = 27 - 2\mu} \qquad \boxed{0 = 2\lambda_2 - 6}$$

$$RHS = \left[6 \times \frac{27}{2}\right] - [27 \times 3] \qquad 2\mu = 27 \qquad 2\lambda_2 = 6$$

$$RHS = 0 = LHS \qquad \boxed{\mu = \frac{27}{2}} \qquad \boxed{\lambda_2 = 3}$$

Q. Find a unit vector perpendicular to each of the vector $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$.

Sol.



Q. Find a unit vector perpendicular to each of the vector $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=3\hat{\imath}+2\hat{\jmath}+2\hat{k}$ and $\vec{b}=\hat{\imath}+2\hat{\jmath}-2\hat{k}$.

Sol. $\vec{a}=3\hat{\imath}+2\hat{\jmath}+2\hat{k}$ $\qquad$ $\vec{b}=\hat{\imath}+2\hat{\jmath}-2\hat{k}$

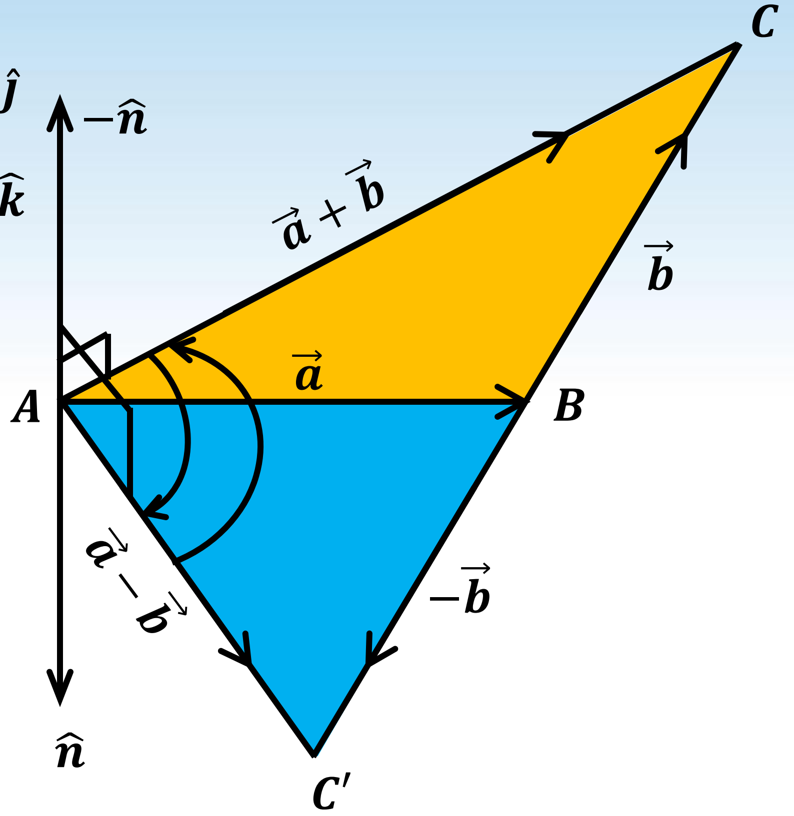
$$\vec{a}+\vec{b}=[3\hat{\imath}+2\hat{\jmath}+2\hat{k}]+[\hat{\imath}+2\hat{\jmath}-2\hat{k}] \quad =4\hat{\imath}+4\hat{\jmath}$$

$$\vec{a}-\vec{b}=[3\hat{\imath}+2\hat{\jmath}+2\hat{k}]-[\hat{\imath}+2\hat{\jmath}-2\hat{k}] \quad =2\hat{\imath}+4\hat{k}$$

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 4 & 4 & 0\\ 2 & 0 & 4\end{vmatrix}$$

$$=\begin{vmatrix}4 & 0\\ 0 & 4\end{vmatrix}\hat{\imath}+\begin{vmatrix}0 & 4\\ 4 & 2\end{vmatrix}\hat{\jmath}+\begin{vmatrix}4 & 4\\ 2 & 0\end{vmatrix}\hat{k}$$

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=16\hat{\imath}-16\hat{\jmath}-8\hat{k}$$



Q. Find a unit vector perpendicular to each of the vector $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=3\hat{\imath}+2\hat{\jmath}+2\hat{k}$ and $\vec{b}=\hat{\imath}+2\hat{\jmath}-2\hat{k}$.

Sol.

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = 16\hat{\imath}-16\hat{\jmath}-8\hat{k} = \vec{n}$$

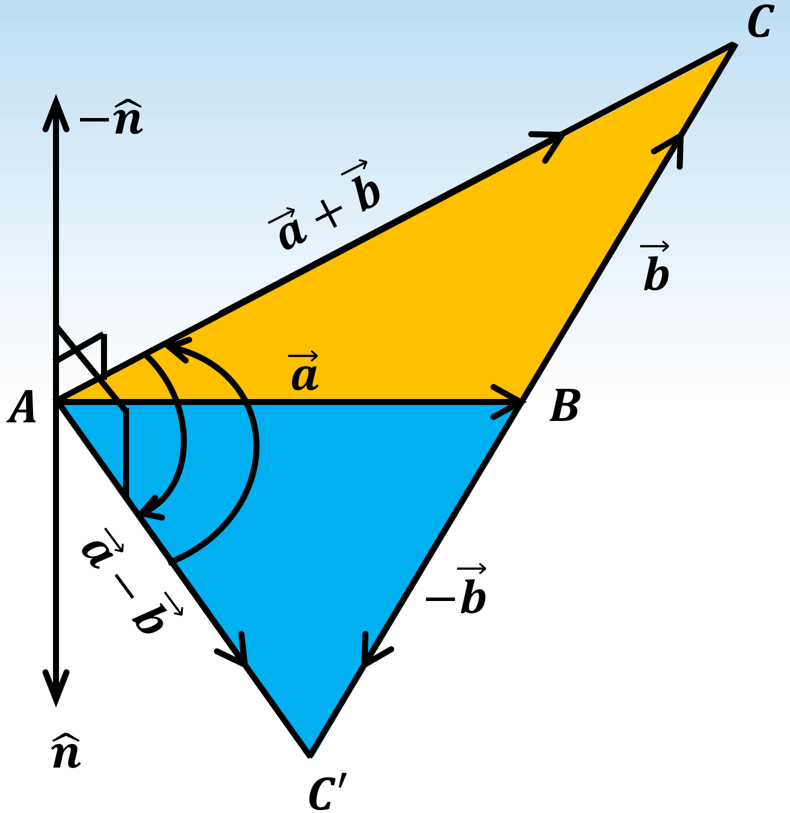
$$|\vec{n}| = \sqrt{16^2+(-16)^2+(-8)^2}$$

$$|\vec{n}| = 24$$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{16\hat{\imath}-16\hat{\jmath}-8\hat{k}}{24}$$

$$\boxed{\hat{n} = \frac{2}{3}\hat{\imath}-\frac{2}{3}\hat{\jmath}-\frac{1}{3}\hat{k}}$$

$$\boxed{-\hat{n} = -\frac{2}{3}\hat{\imath}+\frac{2}{3}\hat{\jmath}+\frac{1}{3}\hat{k}}$$



Summary

Scalar Product

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

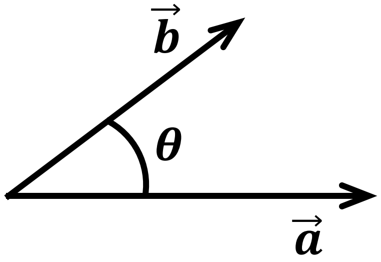
$$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1$$

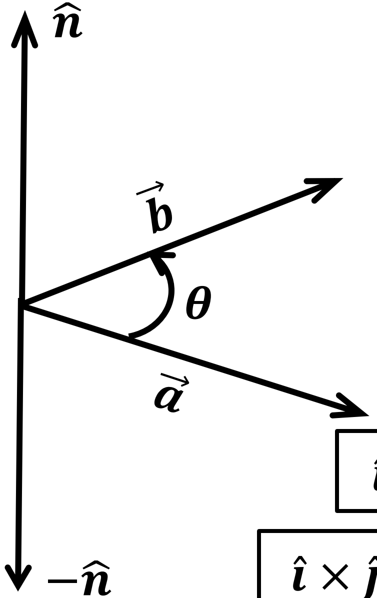
$$\hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{k} = \hat{k}\cdot\hat{\imath} = 0$$

$$\vec{b}\cdot\vec{a} = \vec{a}\cdot\vec{b}$$

$$\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$$

$$(\lambda\vec{a})\cdot\vec{b} = \lambda(\vec{a}\cdot\vec{b}) = \vec{a}\cdot(\lambda\vec{b})$$





Vector Product

$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$$

$$\vec{b}\times\vec{a} = |\vec{b}||\vec{a}|\sin\theta\,(-\hat{n})$$

$$\vec{a}\times\vec{b} = -[\vec{b}\times\vec{a}]$$

$$|\vec{a}\times\vec{b}| = |\vec{b}\times\vec{a}|$$

$$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$$

$$\hat{\imath}\times\hat{\imath} = \hat{\jmath}\times\hat{\jmath} = \hat{k}\times\hat{k} = 0$$

$$\hat{\imath}\times\hat{\jmath} = \hat{k}$$

$$\hat{\jmath}\times\hat{k} = \hat{\imath}$$

$$\hat{k}\times\hat{\imath} = \hat{\jmath}$$

Two Dimensional Space

$P(x_1, y_1)$

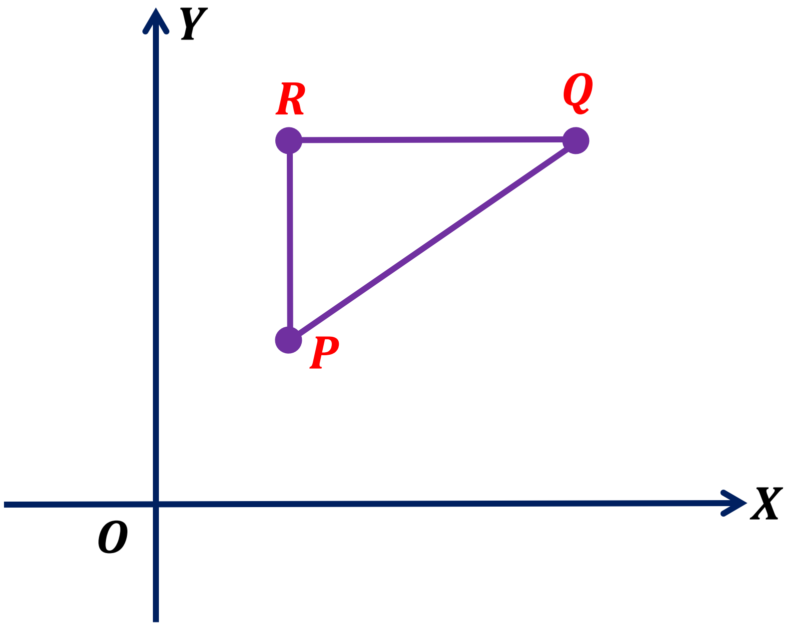
$Q(x_2, y_2)$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$R(x_1, y_2)$

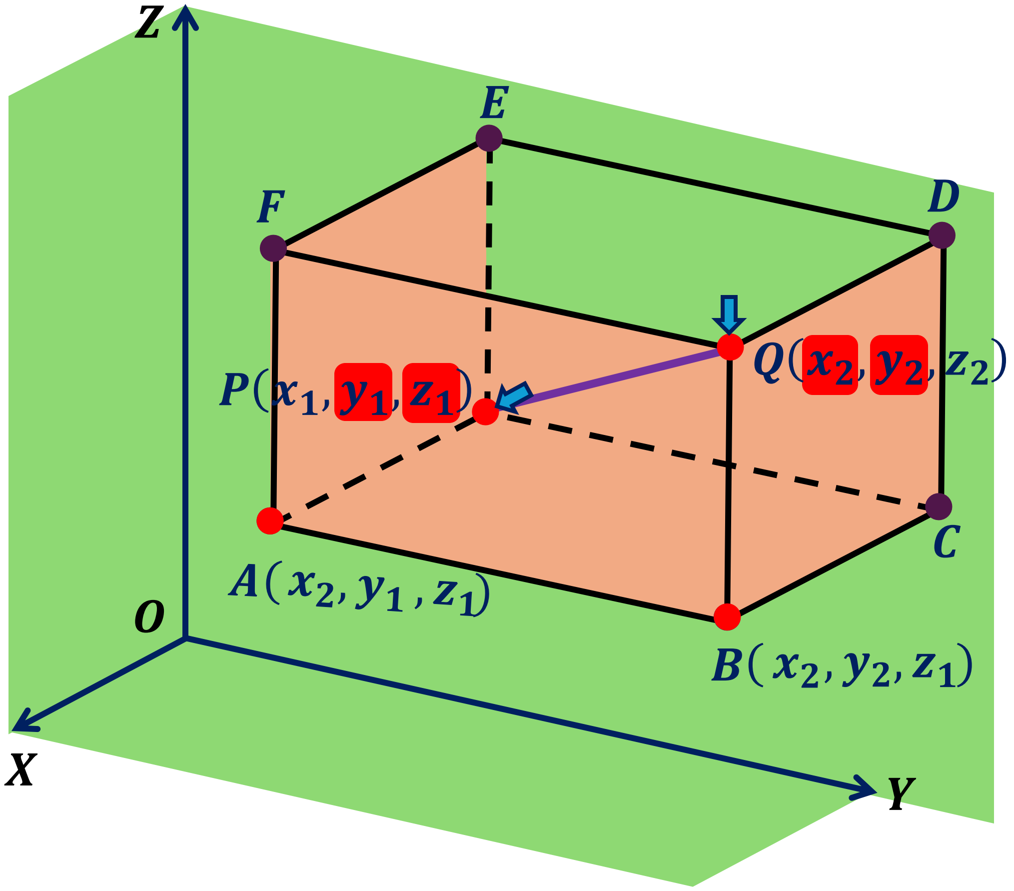
$$RQ = |x_2 - x_1|$$

$$PR = |y_2 - y_1|$$



Three Dimensional Space

$P(x_1, y_1, z_1)$ $Q(x_2, y_2, z_2)$



Three Dimensional Space

$P(x_1, y_1, z_1)$ $Q(x_2, y_2, z_2)$

ΔPAB $PA = |x_2 - x_1|$ $AB = |y_2 - y_1|$

$$PB^2 = PA^2 + AB^2$$

ΔPBQ $BQ = |z_2 - z_1|$

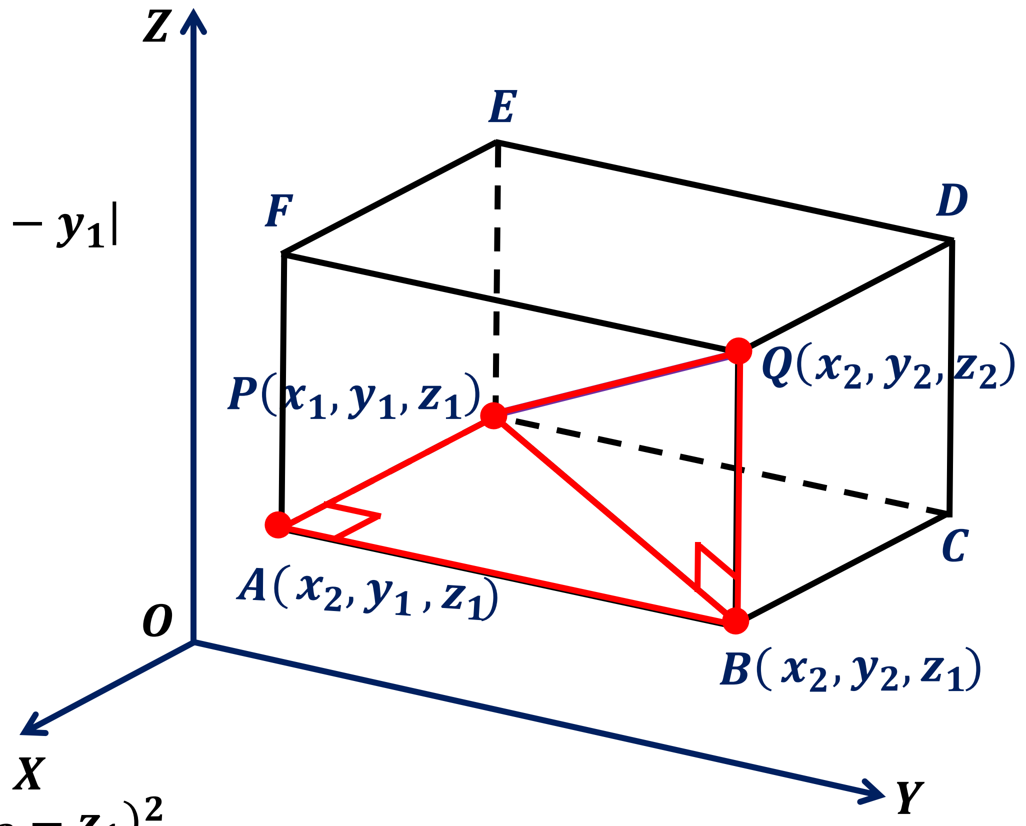
$$PQ^2 = PB^2 + BQ^2$$

$$\Rightarrow PQ^2 = PA^2 + AB^2 + BQ^2$$

$$\Rightarrow PQ^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2 + |z_2 - z_1|^2$$

$$\Rightarrow PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

$$\boxed{PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$



Three Dimensional Space

$O(0,0,0)$ $P(x,y,z)$

$$OP = \sqrt{x^2 + y^2 + z^2}$$

$A \equiv (x,0,0)$ $PA = \sqrt{y^2 + z^2}$

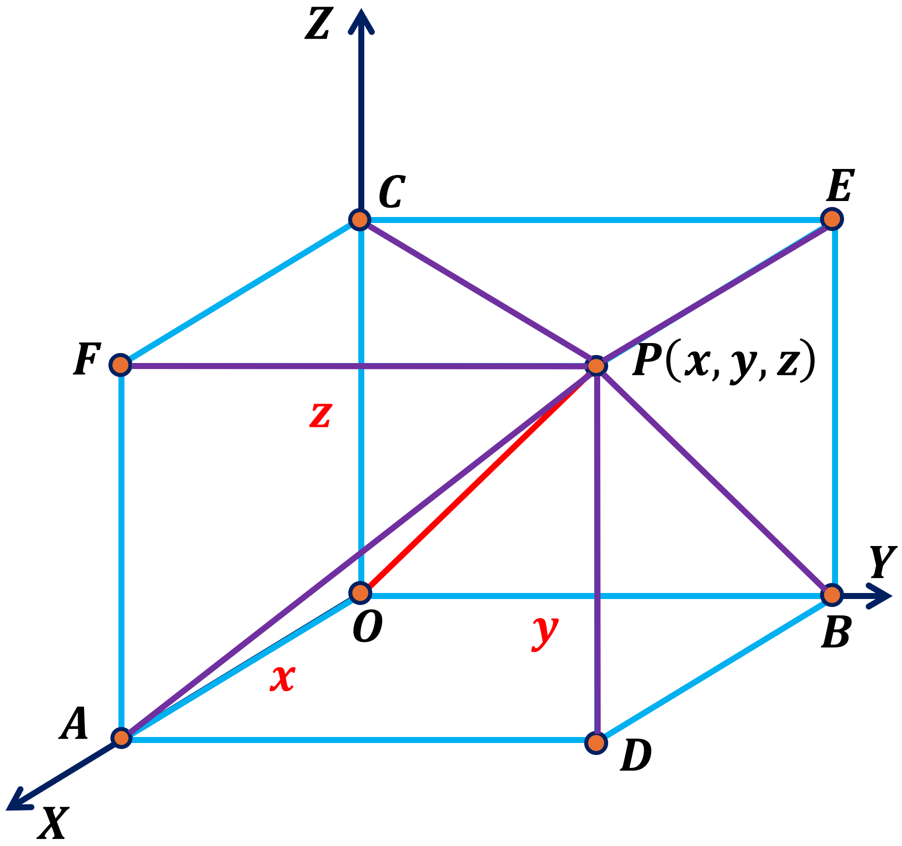
$B \equiv (0,y,0)$ $PB = \sqrt{x^2 + z^2}$

$C \equiv (0,0,z)$ $PC = \sqrt{x^2 + y^2}$

$D \equiv (x,y,0)$ $PD = \sqrt{z^2} = |z|$

$E \equiv (0,y,z)$ $PE = \sqrt{x^2} = |x|$

$F \equiv (x,0,z)$ $PF = \sqrt{y^2} = |y|$



$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Q. Find the distance between the following pairs of points:

(i) $(2, 3, 5)$ and $(4, 3, 1)$ (ii) $(-1, 3, -4)$ and $(1, -3, 4)$

Sol. $P(x_1, y_1, z_1)$ $Q(x_2, y_2, z_2)$ $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

(i) $P(2, 3, 5)$ and $Q(4, 3, 1)$

$x_1 = 2, y_1 = 3, z_1 = 5$

$x_2 = 4, y_2 = 3, z_2 = 1$

$PQ = \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$

$= \sqrt{(2)^2 + (0)^2 + (-4)^2}$

$= \sqrt{4 + 0 + 16}$

$= \sqrt{20}$

$= 2\sqrt{5}$

(ii) $P(-1, 3, -4)$ and $Q(1, -3, 4)$

$x_1 = -1, y_1 = 3, z_1 = -4$

$x_2 = 1, y_2 = -3, z_2 = 4$

PQ

$= \sqrt{\left(1 - (-1)\right)^2 + (-3-3)^2 + \left(4 - (-4)\right)^2}$

$= \sqrt{(2)^2 + (-6)^2 + (8)^2}$

$= \sqrt{4 + 36 + 64}$

$= \sqrt{104} = 2\sqrt{26}$

Q. Show that the points $(-2, 3, 5), (1, 2, 3)$ and $(7, 0, -1)$ are collinear.

Sol. $\boxed{P(-2, 3, 5)}$ $\boxed{Q(1, 2, 3)}$ $\boxed{R(7, 0, -1)}$

Q. Show that the points $(-2,3,5),(1,2,3)$ and $(7,0,-1)$ are collinear.

Sol. $P(-2,3,5)$ $Q(1,2,3)$ $R(7,0,-1)$ **Collinear if they lie on same line**

$$PQ = \sqrt{\left(1-(-2)\right)^2 + (2-3)^2 + (3-5)^2}$$
$$= \sqrt{(3)^2 + (-1)^2 + (-2)^2}$$
$$= \sqrt{9+1+4}$$
$$= \sqrt{14}$$

$$QR = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$$
$$= \sqrt{(6)^2 + (-2)^2 + (-4)^2}$$
$$= \sqrt{36+4+16}$$
$$= \sqrt{56}$$
$$= 2\sqrt{14}$$

PR
$$= \sqrt{\left(7-(-2)\right)^2 + (0-3)^2 + (-1-5)^2}$$
$$= \sqrt{(9)^2 + (-3)^2 + (-6)^2}$$
$$= \sqrt{81+9+36}$$
$$= \sqrt{126}$$
$$= 3\sqrt{14}$$

$$PQ + QR = \sqrt{14} + 2\sqrt{14} = 3\sqrt{14}$$

$$\boxed{PQ + QR = PR}$$

Q. Verify that **points $(-1, 2, 1), (1, -2, 5), (4, -7, 8)$** and **$(2, -3, 4)$** are the **vertices of a parallelogram**.

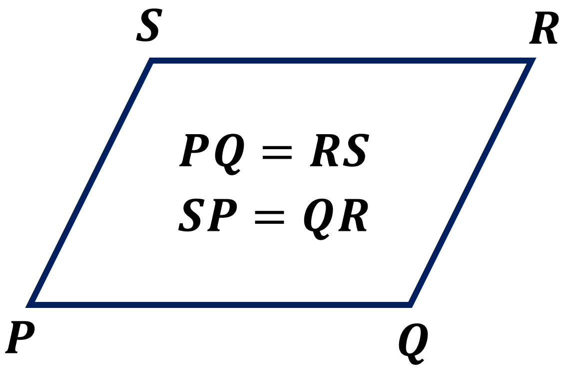
Sol. $\boxed{P(-1, 2, 1)}$ $\boxed{Q(1, -2, 5)}$ $\boxed{R(4, -7, 8)}$ $\boxed{S(2, -3, 4)}$

$$PQ = \sqrt{\left(1-(-1)\right)^2 + (-2-2)^2 + (5-1)^2} = 6$$

$$= \sqrt{(2)^2 + (-4)^2 + (4)^2}$$

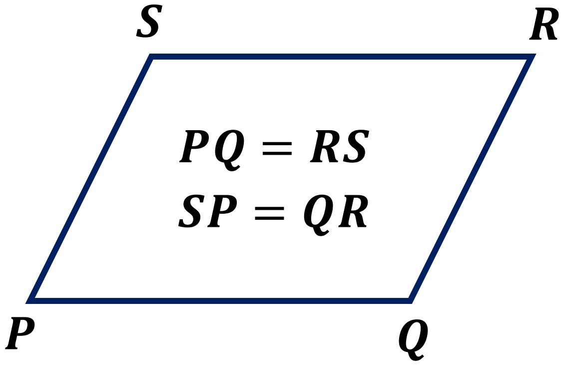
$$= \sqrt{4 + 16 + 16}$$

$$= \sqrt{36}$$



Q. Verify that points $(-1, 2, 1), (1, -2, 5), (4, -7, 8)$ and $(2, -3, 4)$ are the vertices of a parallelogram.

Sol. $P(-1, 2, 1)$ $Q(1, -2, 5)$ $R(4, -7, 8)$ $S(2, -3, 4)$



$$PQ = \sqrt{\left(1-(-1)\right)^2 + (-2-2)^2 + (5-1)^2} = 6$$

$$QR = \sqrt{(4-1)^2 + \left(-7-(-2)\right)^2 + (8-5)^2} = \sqrt{43}$$

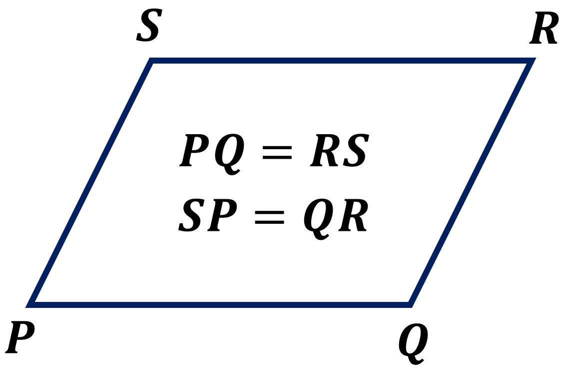
$$= \sqrt{(3)^2 + (-5)^2 + (3)^2}$$

$$= \sqrt{9 + 25 + 9}$$

$$= \sqrt{43}$$

Q. Verify that **points $(-1, 2, 1), (1, -2, 5), (4, -7, 8)$** and **$(2, -3, 4)$** are the **vertices of a parallelogram.**

Sol. $P(-1, 2, 1)$ $Q(1, -2, 5)$ $R(4, -7, 8)$ $S(2, -3, 4)$



$$PQ = \sqrt{\left(1-(-1)\right)^2 + (-2-2)^2 + (5-1)^2} = 6$$

$$QR = \sqrt{(4-1)^2 + \left(-7-(-2)\right)^2 + (8-5)^2} = \sqrt{43}$$

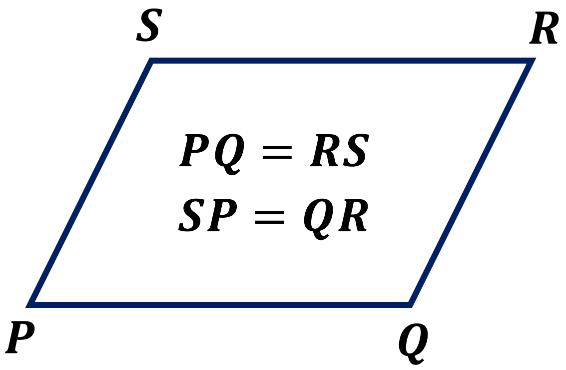
$$RS = \sqrt{(2-4)^2 + \left(-3-(-7)\right)^2 + (4-8)^2} = 6$$

$$= \sqrt{(-2)^2 + (4)^2 + (-4)^2}$$

$$= \sqrt{4 + 16 + 16}$$

$$= \sqrt{36}$$

Q. Verify that **points $(-1, 2, 1), (1, -2, 5), (4, -7, 8)$ and $(2, -3, 4)$ are the vertices of a parallelogram.**



Sol. $P(-1, 2, 1)$ $Q(1, -2, 5)$ $R(4, -7, 8)$ $S(2, -3, 4)$

$$PQ = \sqrt{(1-(-1))^2 + (-2-2)^2 + (5-1)^2} = 6$$

$$QR = \sqrt{(4-1)^2 + (-7-(-2))^2 + (8-5)^2} = \sqrt{43}$$

$$RS = \sqrt{(2-4)^2 + (-3-(-7))^2 + (4-8)^2} = 6$$

$$SP = \sqrt{(-1-2)^2 + (2-(-3))^2 + (1-4)^2} = \sqrt{43}$$

$$= \sqrt{(-3)^2 + (5)^2 + (-3)^2}$$

$$= \sqrt{9 + 25 + 9}$$

$$= \sqrt{43}$$

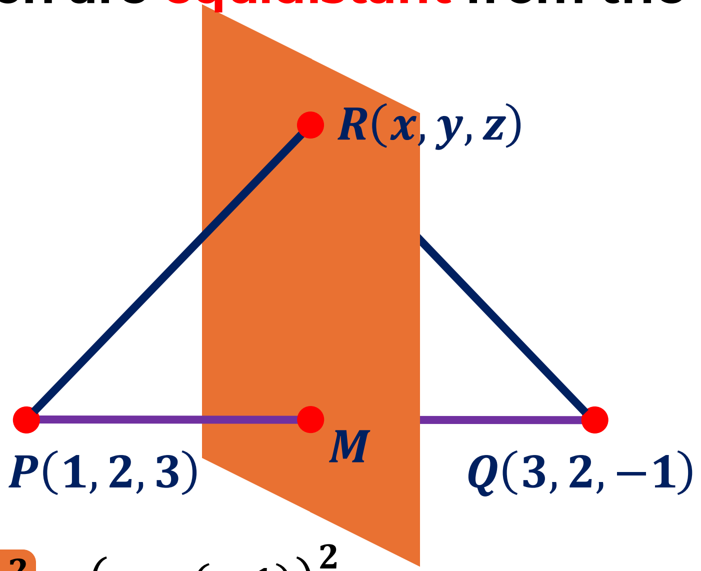
$$\boxed{PQ = RS}$$

$$\boxed{SP = QR}$$

Q. Find the equation of the set of points which are equidistant from the points $(1, 2, 3)$ and $(3, 2, -1)$.

Q. Find the equation of the set of points which are equidistant from the points $(1, 2, 3)$ and $(3, 2, -1)$.

Sol. $\boxed{P(1, 2, 3)}$ $\boxed{Q(3, 2, -1)}$



$$PR = QR$$

$$\sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2}$$

$$= \sqrt{(x-3)^2 + (y-2)^2 + \left(z-(-1)\right)^2}$$

$$(x-1)^2 + \boxed{(y-2)^2} + (z-3)^2 = (x-3)^2 + \boxed{(y-2)^2} + \left(z-(-1)\right)^2$$

$$\Rightarrow (x-1)^2 + (z-3)^2 = (x-3)^2 + (z+1)^2$$

$$\Rightarrow \boxed{x^2} - 2x + \boxed{1} + \boxed{z^2} - 6z + \boxed{9} = \boxed{x^2} - 6x + \boxed{9} + \boxed{z^2} + 2z + \boxed{1}$$

$$\Rightarrow -2x + 6x - 6z - 2z = 0$$

$$\Rightarrow 4x - 8z = 0 \quad \Rightarrow \boxed{x - 2z = 0}$$

Two Dimensional Space

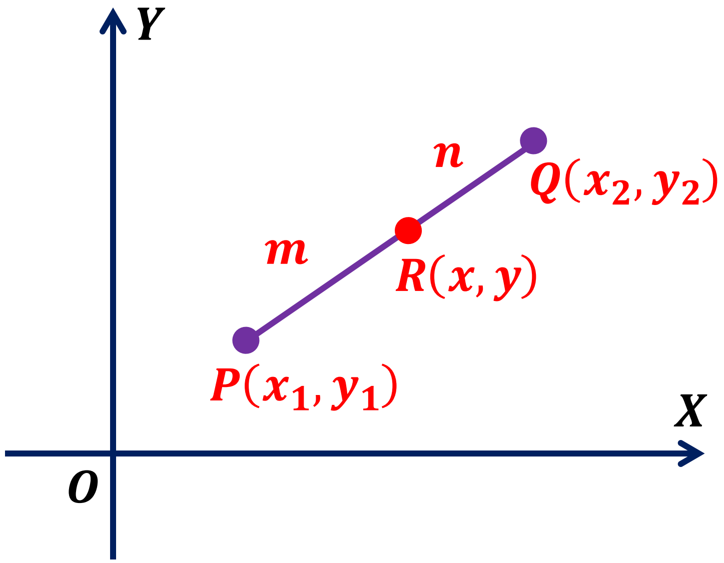
$P(x_1, y_1)$ $Q(x_2, y_2)$ $R(x, y)$

$m : n$ **Internally**

$$R \equiv \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$m : n$ **Externally**

$$R \equiv \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$



R : **Mid-Point**

$$m = n \Rightarrow m : n = 1 : 1$$

$$R \equiv \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

Three Dimensional Space

$P(x_1, y_1, z_1)$ $Q(x_2, y_2, z_2)$ $R(x, y, z)$

$m : n$ **Internally**

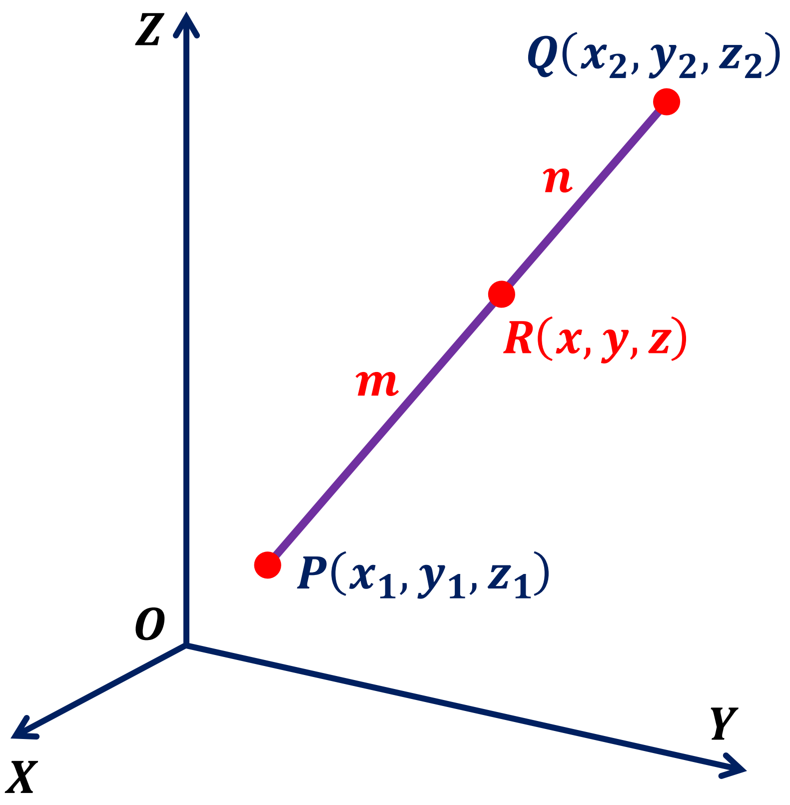
$$R \equiv \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

$m : n$ **Externally**

$$R \equiv \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$

R : **Mid-Point** $m : n = 1 : 1$

$$R \equiv \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}, \frac{z_2 + z_1}{2}\right)$$



Three Dimensional Space

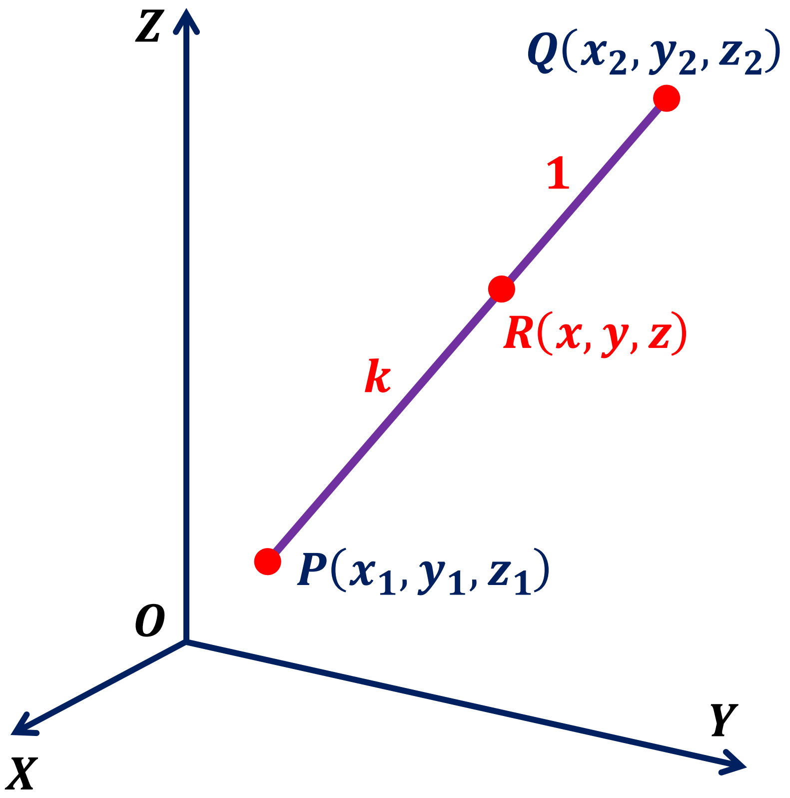
$P(x_1, y_1, z_1)$ | $Q(x_2, y_2, z_2)$ | $R(x, y, z)$

The coordinates of the point R which divides PQ in the ratio $k : 1$.

$$R \equiv \left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}, \frac{kz_2 + z_1}{k+1}\right)$$

$k = +ve$ → **Internally**

$k = -ve$ → **Externally**

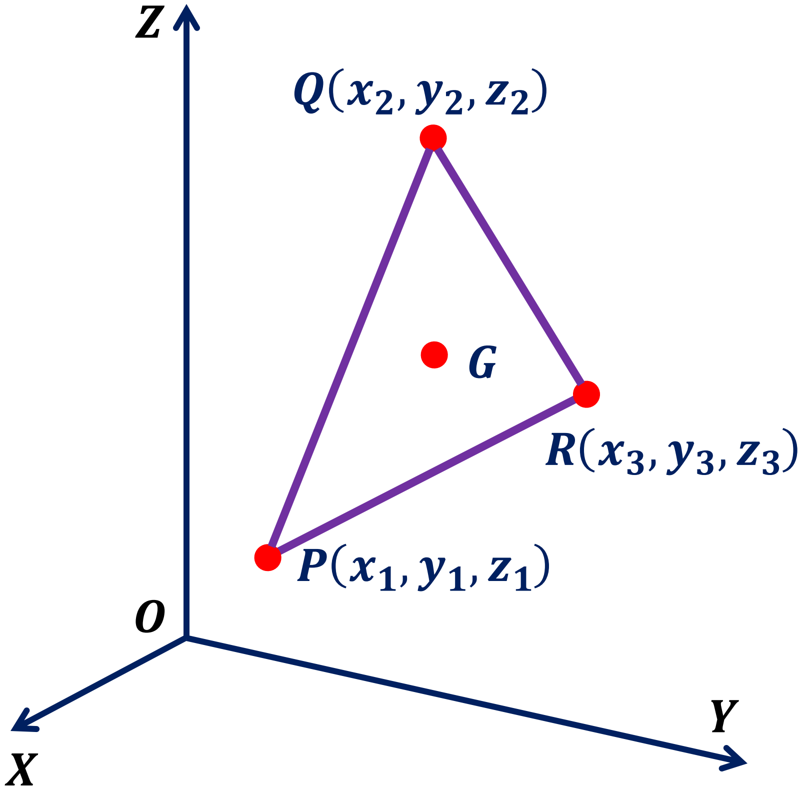


Three Dimensional Space

$P(x_1, y_1, z_1)$ $Q(x_2, y_2, z_2)$ $R(x_3, y_3, z_3)$

G : Centroid

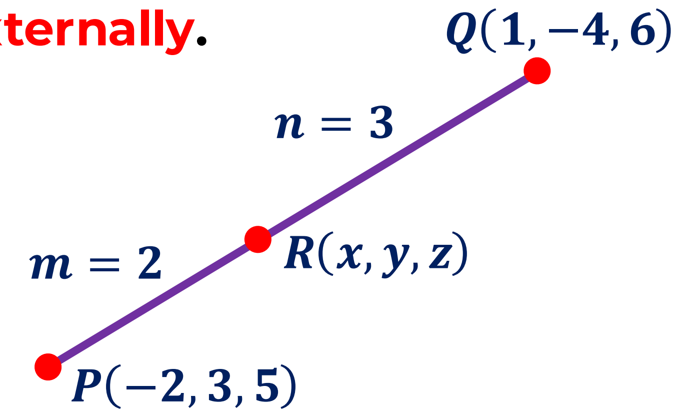
$$G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$



Q. Find the coordinates of the point which divides the line segment joining the points $(-2, 3, 5)$ and $(1, -4, 6)$ in the ratio

(i) $2 : 3$ internally (ii) $2 : 3$ externally.

Sol.



Q. Find the coordinates of the point which divides the line segment joining the points $(-2, 3, 5)$ and $(1, -4, 6)$ in the ratio

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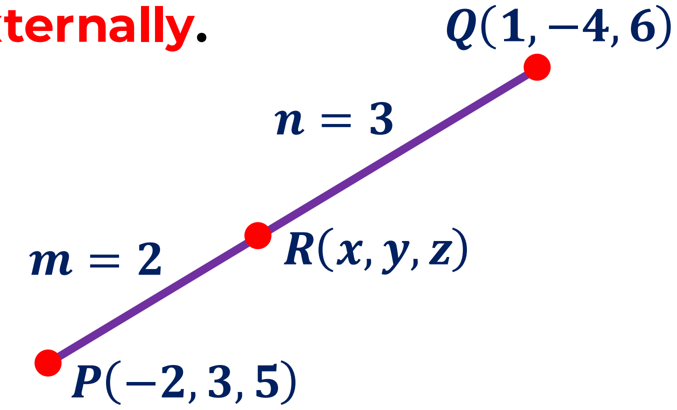


Sol. $\boxed{P(-2, 3, 5)}$ $\boxed{Q(1, -4, 6)}$

(i) $2 : 3$ internally

$x_1 = -2, y_1 = 3, z_1 = 5$

$x_2 = 1, y_2 = -4, z_2 = 6$

$$x = \frac{2(1) + 3(-2)}{2 + 3} = \frac{2 - 6}{5} = -\frac{4}{5}$$

$$y = \frac{2(-4) + 3(3)}{2 + 3} = \frac{-8 + 9}{5} = \frac{1}{5}$$

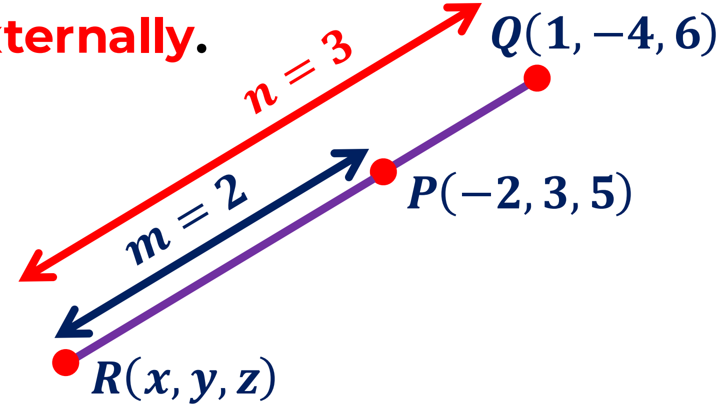
$$z = \frac{2(6) + 3(5)}{2 + 3} = \frac{12 + 15}{5} = \frac{27}{5}$$

$$\boxed{R\left(-\frac{4}{5}, \frac{1}{5}, \frac{27}{5}\right)}$$ $$\boxed{Q(x_2, y_2, z_2)}$$

$$\boxed{R\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}, \frac{mz_2 + nz_1}{m + n}\right)}$$

Q. Find the coordinates of the point which divides the line segment joining the points $(-2, 3, 5)$ and $(1, -4, 6)$ in the ratio

(i) $2 : 3$ internally (ii) $2 : 3$ externally.



Sol. $\boxed{P(-2, 3, 5)}$ $\boxed{Q(1, -4, 6)}$

(ii) $2 : 3$ externally

$x_1 = -2, y_1 = 3, z_1 = 5$

$x_2 = 1, y_2 = -4, z_2 = 6$

$$x = \frac{2(1) - 3(-2)}{2 - 3} = \frac{2 + 6}{-1} = -8$$

$$y = \frac{2(-4) - 3(3)}{2 - 3} = \frac{-8 - 9}{-1} = 17$$

$$z = \frac{2(6) - 3(5)}{2 - 3} = \frac{12 - 15}{-1} = 3$$

$\boxed{R(-8, 17, 3)}$ $\boxed{Q(x_2, y_2, z_2)}$

$$\boxed{R\left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}, \frac{mz_2 - nz_1}{m - n}\right)}$$

Summary

For points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$

Distance	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$	
Coordinates of the point R	**Divides internally in the ratio $m : n$**	$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$
	Divides externally in the ratio $m : n$	$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$
	Mid-point	$\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}, \frac{z_2 + z_1}{2}\right)$

The coordinates of the centroid of the triangle, whose vertices are $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3), are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

Equation of a 3D line

- $\vec{r} = (-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}) + \lambda(-3\hat{\imath} + \hat{\jmath} + 5\hat{k})$

- Passing through point $(-\hat{\imath} + 2\hat{\jmath} + 5\hat{k})$
- And
- Parallel to the line $(-3\hat{\imath} + \hat{\jmath} + 5\hat{k})$

Q. Show that the lines $\vec{r} = \left(-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}\right) + \lambda\left(-3\hat{\imath} + \hat{\jmath} + 5\hat{k}\right)$ and $\vec{r} = \left(-3\hat{\imath} + \hat{\jmath} + 5\hat{k}\right) + \lambda\left(-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}\right)$ are **coplanar**.

Sol. $L_1 : \vec{r} = \left(-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}\right) + \lambda\left(-3\hat{\imath} + \hat{\jmath} + 5\hat{k}\right)$

$$\boxed{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{q}_1 \times \vec{q}_2) = 0}$$

$A(-1, 2, 5) \qquad DR's \rightarrow -3, 1, 5$

$x_1 = -1, y_1 = 2, z_1 = 5$

$a_1 = -3, b_1 = 1, c_1 = 5$

$$\boxed{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0}$$

$L_2 : \vec{r} = \left(-3\hat{\imath} + \hat{\jmath} + 5\hat{k}\right) + \lambda\left(-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}\right)$

$B(-3, 1, 5) \qquad DR's \rightarrow -1, 2, 5$

$x_2 = -3, y_2 = 1, z_2 = 5$

$a_2 = -1, b_2 = 2, c_2 = 5$

$$\begin{vmatrix} -3-(-1) & 1-2 & 5-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = \begin{vmatrix} -2 & -1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$$

$$= -2(5-10) - (-1)\left(-15-(-5)\right) + 0$$

$$= 10 - 10 + 0$$

$$= 0$$

Q. **Find the distance of a point $(3, -2, 1)$ from the plane**

$$\vec{r} \cdot \left(-2\hat{\imath} + \hat{\jmath} - 2\hat{k}\right) = 3$$

Sol.

Q. Find the distance of a point $(3, -2, 1)$ from the plane $\vec{r} \cdot \left(-2\hat{i} + \hat{j} - 2\hat{k}\right) = 3$

Sol. $A(3, -2, 1)$

$$\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$$

$$\pi : \vec{r} \cdot \left(-2\hat{i} + \hat{j} - 2\hat{k}\right) = 3$$

$$d = 3 \qquad \vec{n} = -2\hat{i} + \hat{j} - 2\hat{k}$$

$$\boxed{\pi : \vec{r} \cdot \vec{n} = d}$$

$$\boxed{A(\vec{a})}$$

$$\boxed{\frac{|\vec{a} \cdot \vec{n} - d|}{|\vec{n}|}}$$

$$\begin{aligned} \vec{a} \cdot \vec{n} &= \left(3\hat{i} - 2\hat{j} + \hat{k}\right) \cdot \left(-2\hat{i} + \hat{j} - 2\hat{k}\right) \\ &= -6 - 2 - 2 \\ &= -10 \end{aligned}$$

$$|\vec{n}| = \sqrt{(-2)^2 + 1^2 + (-2)^2} = 3$$

$$\text{Distance} = \frac{|-10 - 3|}{3}$$

$$\boxed{\text{Distance} = \frac{13}{3}}$$

Q. Find the distance of the point $(2, 3, -5)$ from the plane $x + 2y - 2z = 9$.

Sol.

Q. Find the **distance** of the **point** $(2, 3, -5)$ from the **plane** $x + 2y - 2z = 9$.

Sol. $P(2, 3, -5)$

$x_1 = 2, y_1 = 3, z_1 = -5$

$\pi : x + 2y - 2z = 9$

$A = 1, B = 2, C = -2, D = 9$

$$\boxed{\pi : Ax + By + Cz = D}$$

$$\boxed{P(x_1, y_1, z_1)}$$

$$\boxed{d = \left|\frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A_1^2 + B_1^2 + C_1^2}}\right|}$$

$$d = \left|\frac{1(2) + 2(3) + (-2)(-5) - 9}{\sqrt{1^2 + 2^2 + (-2)^2}}\right|$$

$$\Rightarrow d = \left|\frac{2 + 6 + 10 - 9}{\sqrt{1 + 4 + 4}}\right|$$

$$\Rightarrow d = \left|\frac{9}{\sqrt{9}}\right|$$

$$\Rightarrow \boxed{d = 3}$$

Angle between the Planes

= Angle between the Normals

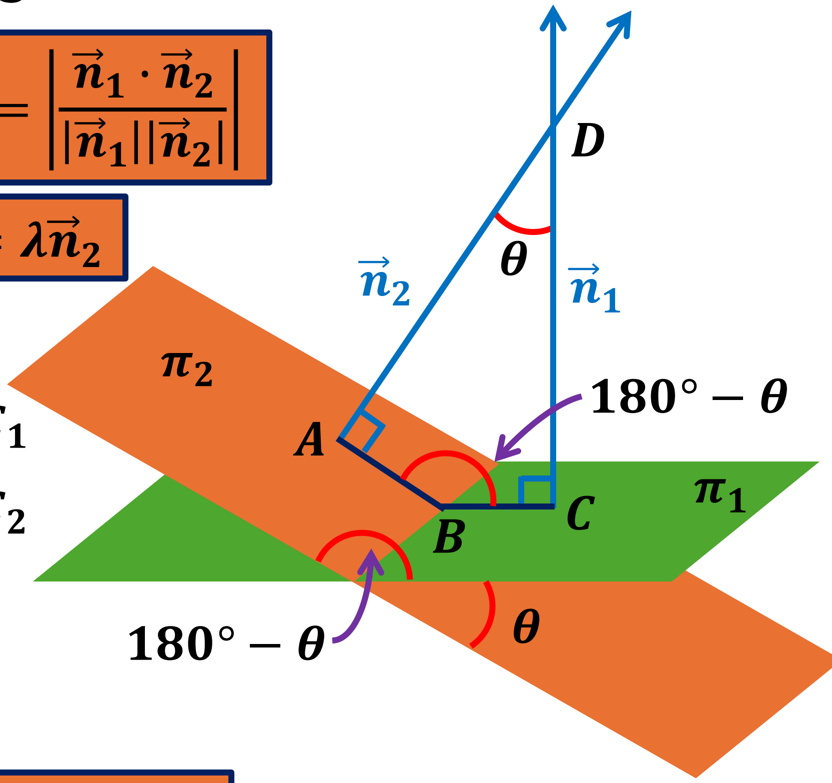
$\vec{n}_1 \rightarrow$ **Normal Vector of π_1 Plane**

$\vec{n}_2 \rightarrow$ **Normal Vector of π_2 Plane**

$$\pi_1 : \vec{r} \cdot \vec{n}_1 = d_1 \qquad \pi_2 : \vec{r} \cdot \vec{n}_2 = d_2$$

$$\boxed{\cos\theta = \left|\frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|}$$

$$\boxed{\text{Perpendicular} : \vec{n}_1 \cdot \vec{n}_2 = 0} \qquad \boxed{\text{Parallel} : \vec{n}_1 = \lambda \vec{n}_2}$$



Cartesian Form

$$\pi_1 : A_1x + B_1y + C_1z + D_1 = 0 \quad DR's \rightarrow A_1, B_1, C_1$$

$$\pi_2 : A_2x + B_2y + C_2z + D_2 = 0 \quad DR's \rightarrow A_2, B_2, C_2$$

$$\boxed{\cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}}\right|}$$

$$\boxed{\begin{array}{c}\text{Perpendicular} \\ A_1A_2 + B_1B_2 + C_1C_2 = 0\end{array}} \qquad \boxed{\text{Parallel} : \frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}}$$

Q. Find the **angle** between the **planes** whose **vector equations** are $\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})=5$ and $\vec{r}\cdot(3\hat{i}-3\hat{j}+5\hat{k})=3$.

Sol. $\pi_1 : \vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})=5$ $\quad \pi_2 : \vec{r}\cdot(3\hat{i}-3\hat{j}+5\hat{k})=3$

$\vec{n}_1 = 2\hat{i}+2\hat{j}-3\hat{k}$ $\quad \vec{n}_2 = 3\hat{i}-3\hat{j}+5\hat{k}$

$$\boxed{\pi_1 : \vec{r}\cdot\vec{n}_1 = d_1}$$

$$\boxed{\pi_2 : \vec{r}\cdot\vec{n}_2 = d_2}$$

$$\vec{n}_1\cdot\vec{n}_2 = (2\hat{i}+2\hat{j}-3\hat{k})\cdot(3\hat{i}-3\hat{j}+5\hat{k})$$
$$= 6-6-15$$
$$= -15$$

$$\boxed{\cos\theta = \left|\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|}$$

$$|\vec{n}_1| = \sqrt{2^2+2^2+(-3)^2} = \sqrt{17}$$
$$|\vec{n}_2| = \sqrt{3^2+(-3)^2+5^2} = \sqrt{43}$$

$$\cos\theta = \left|\frac{-15}{\sqrt{17}\sqrt{43}}\right|$$

$$\Rightarrow \cos\theta = \frac{15}{\sqrt{731}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{15}{\sqrt{731}}\right)}$$

Q. Find the angle between the two planes $4x + 8y + z - 8 = 0$ and $y + z - 4 = 0$.

Sol. $\pi_1 : 4x + 8y + z - 8 = 0$

$A_1 = 4, B_1 = 8, C_1 = 1$

$\pi_2 : y + z - 4 = 0$

$\pi_2 : 0x + y + z - 4 = 0$

$A_2 = 0, B_2 = 1, C_2 = 1$

$$\boxed{\pi_1 : A_1x + B_1y + C_1z + D_1 = 0}$$

$$\boxed{\pi_2 : A_2x + B_2y + C_2z + D_2 = 0}$$

$$\boxed{\cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}}\right|}$$

$$\cos\theta = \left|\frac{4(0) + 8(1) + 1(1)}{\sqrt{4^2 + 8^2 + 1^2}\sqrt{0^2 + 1^2 + 1^2}}\right|$$

$$\Rightarrow \cos\theta = \left|\frac{9}{\sqrt{81}\sqrt{2}}\right|$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \boxed{\theta = 45^\circ}$$

$\pi : \vec{r} \cdot \vec{n} = d$

$L : \vec{r} = \vec{a} + \lambda \vec{q}$

$$\cos\theta = \left| \frac{\vec{q} \cdot \vec{n}}{|\vec{q}||\vec{n}|} \right|$$

The angle between a line and a plane is the complement of the angle between the line and normal to the plane.

$\phi \rightarrow$ Angle between Line and Plane

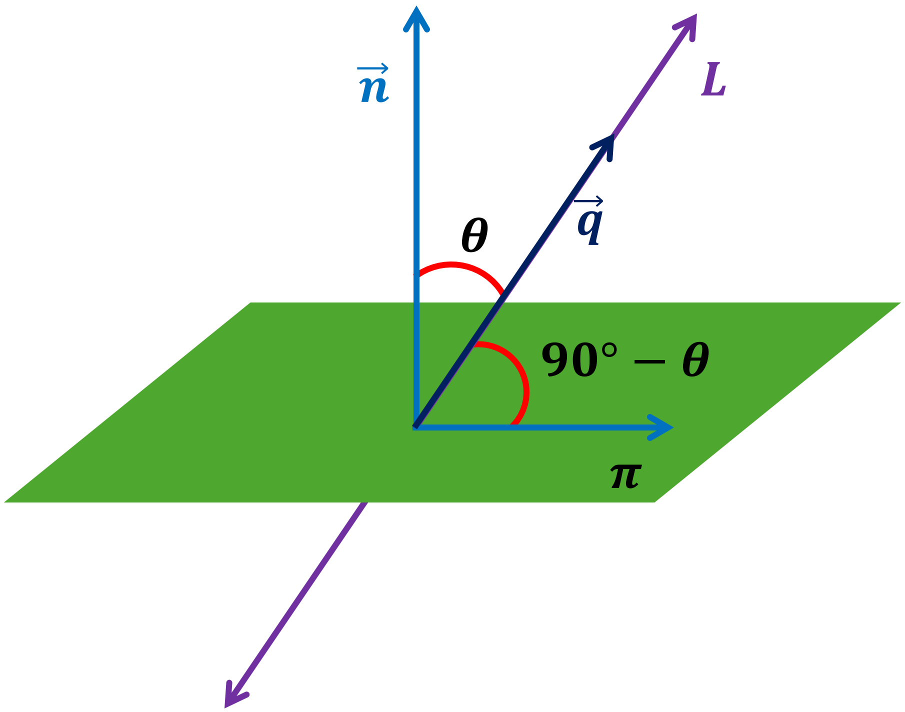
$$\phi = 90^\circ - \theta$$

$$\Rightarrow \sin\phi = \sin(90^\circ - \theta)$$

$$\Rightarrow \sin\phi = \cos\theta$$

$$\Rightarrow \sin\phi = \left| \frac{\vec{q} \cdot \vec{n}}{|\vec{q}||\vec{n}|} \right|$$

$$\boxed{\phi = \sin^{-1}\left(\left| \frac{\vec{q} \cdot \vec{n}}{|\vec{q}||\vec{n}|} \right| \right)}$$



Q. Find the angle between the line

$$\frac{x-1}{10}=\frac{y+2}{2}=\frac{z-4}{-11}$$

and the plane $2x+3y+6z-12=0$.

$$L:\vec{r}=\vec{a}+\lambda\vec{q}$$

$$\pi:\vec{r}\cdot\vec{n}=d$$

$$\phi=\sin^{-1}\left(\left|\frac{\vec{q}\cdot\vec{n}}{|\vec{q}||\vec{n}|}\right|\right)$$

Sol. $L:\dfrac{x-1}{10}=\dfrac{y+2}{2}=\dfrac{z-4}{-11}$

$L:\dfrac{x-1}{10}=\dfrac{y-(-2)}{2}=\dfrac{z-4}{-11}$

$DR's\rightarrow 10,2,-11$

$\vec{q}=10\hat{\imath}+2\hat{\jmath}-11\hat{k}$

$\pi:2x+3y+6z-12=0$

$DR's\rightarrow 2,3,6$

$\vec{n}=2\hat{\imath}+3\hat{\jmath}+6\hat{k}$

$$\vec{q}\cdot\vec{n}=\left(10\hat{\imath}+2\hat{\jmath}-11\hat{k}\right)\cdot\left(2\hat{\imath}+3\hat{\jmath}+6\hat{k}\right)$$

$$=20+6-66$$

$$=-40$$

$$|\vec{q}|=\sqrt{10^2+2^2+(-11)^2}=15$$

$$|\vec{n}|=\sqrt{2^2+3^2+6^2}=7$$

$$\phi=\sin^{-1}\left(\left|\frac{-40}{15\times 7}\right|\right)$$

$$\boxed{\phi=\sin^{-1}\left(\frac{8}{21}\right)}$$

Summary

- Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{q}_1$ and $\vec{r} = \vec{a}_2 + \lambda\vec{q}_2$ are coplanar if

$$\boxed{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{q}_1 \times \vec{q}_2) = 0}$$

- Two lines

$\dfrac{x - x_1}{a_1} = \dfrac{y - y_1}{b_1} = \dfrac{z - z_1}{c_1}$ and $\dfrac{x - x_2}{a_2} = \dfrac{y - y_2}{b_2} = \dfrac{z - z_2}{c_2}$ are coplaner if

$$\boxed{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0}$$

- If θ is the angle between the two planes $\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$, then

$$\boxed{\theta = \cos^{-1}\left|\frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|}$$

Summary

- The angle ϕ between the line $\vec{r} = \vec{a} + \lambda\vec{q}$ and the plane $\vec{r} \cdot \vec{n} = d$ is

$$\boxed{\phi = \sin^{-1}\left|\frac{\vec{q} \cdot \vec{n}}{|\vec{q}||\vec{n}|}\right|}$$

- The angle θ between the planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is given by

$$\cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}}\right|$$

- The distance of a point whose position vector is $\vec{a}$ from the plane $\vec{r} \cdot \hat{n} = d$ is $|d - \vec{a} \cdot \hat{n}|$.
- The distance from a point (x_1, y_1, z_1) to the plane $Ax + By + Cz + D = 0$ is

$$\left|\frac{Ax_1 + By_1 + Cz_1}{\sqrt{A^2 + B^2 + C^2}}\right|$$