Math at a Glance

A Collection of Timeless Mathematical Formulae

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Trigonometry

$$\sin^2 x + \cos^2 x = 1$$
$$\sec^2 x - \tan^2 x = 1$$
$$\csc^2 x - \cot^2 x = 1$$
$$\sin^2 \frac{x}{2} = \frac{1-\cos x}{2}$$
$$\cos^2 \frac{x}{2} = \frac{1+\cos x}{2}$$
$$\tan^2 \frac{x}{2} = \frac{1-\cos x}{1+\cos x}$$
$$\sin(2x) = 2\sin x\cos x$$
$$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$$
$$\sin(3x) = 3\sin x - 4\sin^3 x$$
$$\cos(3x) = 4\cos^3 x - 3\cos x$$
$$\tan(3x) = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$
$$\sin(x+y) = \sin x\cos y + \cos x\sin y$$
$$\cos(x+y) = \cos x\cos y - \sin x\sin y$$
$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$$
$$\sin(x-y) = \sin x\cos y - \cos x\sin y$$
$$\cos(x-y) = \cos x\cos y + \sin x\sin y$$
$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$$
$$2\sin x\sin y = \cos(x-y) - \cos(x+y)$$
$$2\cos x\cos y = \cos(x-y) + \cos(x+y)$$
$$2\sin x\cos y = \sin(x+y) + \sin(x-y)$$
$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$
$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$
$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\tanh x = \frac{\sinh x}{\cosh x}$$
$$\coth x = \frac{\cosh x}{\sinh x}$$
$$sech\, x = \frac{1}{\cosh x}$$
$$csch\, x = \frac{1}{\sinh x}$$

Differentiation

$$D_x[c] = 0$$
$$D_x[x] = 1$$
$$D_x[kx] = k$$
$$D_x[x^n] = nx^{n-1}$$
$$D_x[\sqrt{x}] = \frac{1}{2\sqrt{x}}$$
$$D_x[x^{1/n}] = \frac{1}{n}x^{\frac{1}{n}-1}$$
$$D_x[|x|] = \frac{x}{|x|}, \quad x \neq 0$$
$$D_x[f(g(x))] = f'(g(x))g'(x)$$
$$D_x[e^x] = e^x$$
$$D_x[a^x] = a^x \ln a$$
$$D_x[e^{f(x)}] = f'(x)e^{f(x)}$$
$$D_x[a^{f(x)}] = f'(x)a^{f(x)}\ln a$$
$$D_x[\ln x] = \frac{1}{x}$$
$$D_x[\log_a x] = \frac{1}{x\ln a}$$
$$D_x[\ln f(x)] = \frac{f'(x)}{f(x)}$$
$$D_x[\sin x] = \cos x$$
$$D_x[\cos x] = -\sin x$$
$$D_x[\tan x] = \sec^2 x$$
$$D_x[\cot x] = -\csc^2 x$$
$$D_x[\sec x] = \sec x\tan x$$
$$D_x[\csc x] = -\csc x\cot x$$
$$D_x[\sin(ax)] = a\cos(ax)$$
$$D_x[\cos(ax)] = -a\sin(ax)$$
$$D_x[\tan(ax)] = a\sec^2(ax)$$
$$D_x[\cot(ax)] = -a\csc^2(ax)$$
$$D_x[\sec(ax)] = a\sec(ax)\tan(ax)$$
$$D_x[\csc(ax)] = -a\csc(ax)\cot(ax)$$
$$D_x[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$$
$$D_x[\cos^{-1} x] = -\frac{1}{\sqrt{1-x^2}}$$
$$D_x[\tan^{-1} x] = \frac{1}{1+x^2}$$
$$D_x[\cot^{-1} x] = -\frac{1}{1+x^2}$$
$$D_x[\sec^{-1} x] = \frac{1}{|x|\sqrt{x^2-1}}$$
$$D_x[\csc^{-1} x] = -\frac{1}{|x|\sqrt{x^2-1}}$$

Integration

$$x^n \mapsto \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$
$$\frac{1}{x} \mapsto \ln|x| + C$$
$$e^x \mapsto e^x + C$$
$$a^x \mapsto \frac{a^x}{\ln a} + C$$
$$\ln x \mapsto x\ln x - x + C$$
$$\sin x \mapsto -\cos x + C$$
$$\sin^2 x \mapsto \frac{x}{2} - \frac{\sin 2x}{4} + C$$
$$\sin(ax) \mapsto -\frac{1}{a}\cos(ax) + C$$
$$\cos x \mapsto \sin x + C$$
$$\cos^2 x \mapsto \frac{x}{2} + \frac{\sin 2x}{4} + C$$
$$\cos(ax) \mapsto \frac{1}{a}\sin(ax) + C$$
$$\tan x \mapsto \ln|\sec x| + C$$
$$\tan^2 x \mapsto \tan x - x + C$$
$$\csc x \mapsto \ln|\csc x - \cot x| + C$$
$$\csc^2 x \mapsto -\cot x + C$$
$$\sec x \mapsto \ln|\sec x + \tan x| + C$$
$$\sec^2 x \mapsto \tan x + C$$
$$\cot x \mapsto \ln|\sin x| + C$$
$$\cot^2 x \mapsto -\cot x - x + C$$
$$\sec x\tan x \mapsto \sec x + C$$
$$\csc x\cot x \mapsto -\csc x + C$$
$$\sinh x \mapsto \cosh x + C$$
$$\cosh x \mapsto \sinh x + C$$
$$\tanh x \mapsto \ln(\cosh x) + C$$
$$\coth x \mapsto \ln|\sinh x| + C$$
$$\mathrm{sech}^2 x \mapsto \tanh x + C$$
$$\mathrm{csch}^2 x \mapsto -\coth x + C$$
$$\mathrm{sech}\, x \mapsto \arctan(\sinh x) + C$$
$$\mathrm{csch}\, x \mapsto \ln\left|\tanh\frac{x}{2}\right| + C$$
$$\frac{1}{1+x^2} \mapsto \arctan x + C$$
$$\frac{1}{\sqrt{1-x^2}} \mapsto \arcsin x + C$$

Trigonometry

$$\sinh 2x = 2\sinh x\cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$$
$$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$$
$$\sinh^2 \frac{x}{2} = \frac{\cosh x - 1}{2}$$
$$\cosh^2 \frac{x}{2} = \frac{\cosh x + 1}{2}$$
$$\tanh^2 \frac{x}{2} = \frac{\cosh x - 1}{\cosh x + 1}$$
$$\coth^2 \frac{x}{2} = \frac{\cosh x + 1}{\cosh x - 1}$$
$$\sin x\cos x = \frac{1}{2}\sin 2x$$
$$1 - \sin x = 2\sin^2 \frac{\pi/4 - x/2}{}$$
$$1 + \sin x = 2\cos^2 \frac{\pi/4 - x/2}{}$$
$$\sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$$
$$\sin x - \cos x = \sqrt{2}\cos\left(x + \frac{\pi}{4}\right)$$

Differentiation

$$D_x[\sinh x] = \cosh x$$
$$D_x[\cosh x] = \sinh x$$
$$D_x[\tanh x] = \operatorname{sech}^2 x$$
$$D_x[\coth x] = -\operatorname{csch}^2 x$$
$$D_x[\operatorname{sech} x] = -\operatorname{sech} x\tanh x$$
$$D_x[\operatorname{csch} x] = -\operatorname{csch} x\coth x$$
$$D_x[\sinh^{-1} x] = \frac{1}{\sqrt{x^2+1}}$$
$$D_x[\cosh^{-1} x] = \frac{1}{\sqrt{x^2-1}}$$
$$D_x[\tanh^{-1} x] = \frac{1}{1-x^2}$$
$$D_x[\coth^{-1} x] = \frac{1}{1-x^2}$$

Integration

$$\frac{1}{\sqrt{a^2-x^2}} \mapsto \arcsin\left(\tfrac{x}{a}\right) + C$$
$$\frac{-1}{\sqrt{a^2-x^2}} \mapsto \arccos\left(\tfrac{x}{a}\right) + C$$
$$\frac{1}{a^2+x^2} \mapsto \tfrac{1}{a}\arctan\left(\tfrac{x}{a}\right) + C$$
$$\frac{-1}{a^2+x^2} \mapsto -\tfrac{1}{a}\arctan\left(\tfrac{x}{a}\right) + C$$
$$\frac{1}{\sqrt{x^2+a^2}} \mapsto \operatorname{arcsinh}\left(\tfrac{x}{a}\right) + C$$
$$\frac{1}{\sqrt{x^2-a^2}} \mapsto \operatorname{arccosh}\left(\tfrac{x}{a}\right) + C$$
$$\frac{1}{x\sqrt{x^2-1}} \mapsto \operatorname{arcsec} x + C$$
$$\frac{1}{x\sqrt{a^2-x^2}} \mapsto -\tfrac{1}{a}\operatorname{arccsc}\left(\tfrac{a}{x}\right) + C$$

Euler's Formulae

$e^{i\pi} + 1 = 0$ (Euler's identity)

$e^{i\theta} = \cos\theta + i\sin\theta$ (Euler's Formula)

Fresnel Sine

$$S(x) = \int_0^x \sin\left(\tfrac{\pi}{2}t^2\right)\,dt$$

Series & Sequences

Arithmetic sum: $S_n = \frac{n}{2}(a_1 + a_n)$
Geometric sum: $S_n = a\frac{1-r^n}{1-r}$
Infinite GP: $S_\infty = \frac{a}{1-r}, |r| < 1$
Harmonic number: $H_n = \sum_{k=1}^{n} \frac{1}{k}$
Convergence test: $\sum \frac{1}{n^p}$ converges if $p > 1$

Vector Calculus

$\nabla f = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z})$
Divergence: $\nabla \cdot \vec{F} = \frac{\partial F_x}{x} + \frac{\partial F_y}{y} + \frac{\partial F_z}{z}$
Curl: $\nabla \times \vec{F} = \det \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial_x & \partial_y & \partial_z \\ F_x & F_y & F_z \end{bmatrix}$
Green's theorem: $\oint_C (Pdx + Qdy) = \iint_R (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})dA$

Complex Numbers

$z = a + bi = r(\cos\theta + i\sin\theta)$
$r = |z| = \sqrt{a^2+b^2}, \theta = \tan^{-1}(b/a)$
De Moivre: $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$
Roots of unity: $z_k = e^{2\pi ik/n},\ k = 0, \ldots, n-1$

Differential Equations

$y' = ky \quad \Rightarrow \quad y = Ce^{kt}$
$y'' + ay' + by = 0 \quad \Rightarrow \quad e^{\lambda t}, \lambda^2 + a\lambda + b = 0$

Transforms

Fourier: $\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)e^{-2\pi ix\xi}\,dx$
Inverse: $f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi)e^{2\pi ix\xi}\,d\xi$
Laplace: $\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt$
Laplace: $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$
Laplace: $\mathcal{L}\{\sin bt\} = \frac{b}{s^2+b^2}$

Reduction Formula

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}I_{n-2}$$
$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\,\sin x + \frac{n-1}{n}I_{n-2}$$
$$\int \tan^n x\,dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x\,dx$$
$$\int \csc^n x\,dx = -\frac{1}{n-1}\csc^{n-1} x\cot x + \frac{n-2}{n-1}\int \csc^{n-2} x\,dx$$
$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-1} x\tan x + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$$
$$\int \cot^n x\,dx = -\frac{1}{n-1}\cot^{n-1} x - \int \cot^{n-2} x\,dx$$

Series Expansions

$$\sin x = x - \tfrac{x^3}{3!} + \tfrac{x^5}{5!} - \tfrac{x^7}{7!} + \ldots$$
$$\cos x = 1 - \tfrac{x^2}{2!} + \tfrac{x^4}{4!} - \tfrac{x^6}{6!} + \ldots$$
$$\tan x = x + \tfrac{x^3}{3} + \tfrac{2x^5}{15} + \tfrac{17x^7}{315} + \ldots$$
$$e^x = 1 + \tfrac{x}{1!} + \tfrac{x^2}{2!} + \tfrac{x^3}{3!} + \ldots$$
$$a^x = 1 + \tfrac{x}{1!}(lna) + \tfrac{x^2}{2!}(lna)^2 + \ldots$$
$$\sinh x = x + \tfrac{x^3}{3!} + \tfrac{x^5}{5!} + \ldots$$
$$\cosh x = 1 + \tfrac{x^2}{2!} + \tfrac{x^4}{4!} + \tfrac{x^6}{6!} + \ldots$$
$$\sin^{-1} x = x + (1^2)\tfrac{x^3}{3!} + (1^2.3^2)\tfrac{x^5}{5!} + \ldots$$
$$\tan^{-1} x = x + \tfrac{1}{3}x^3 + \tfrac{1}{5}x^5 - \ldots$$
$$\sec^{-1} x = 1 + \tfrac{x^2}{2!} + 5.\tfrac{x^4}{4!} + 61.\tfrac{x^6}{6!} + \ldots$$
$$\ln(1+x) = x - \tfrac{x^2}{2} + \tfrac{x^3}{3} - \tfrac{x^4}{4} + \ldots$$
$$(1+x)^n = 1 + nx + \tfrac{n(n-1)}{2!}x^2 + \ldots$$

Half-angle Formula

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}},$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}},$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}.$$

$$\sec\frac{\theta}{2} = \pm\sqrt{\frac{2}{1+\cos\theta}},$$

$$\csc\frac{\theta}{2} = \pm\sqrt{\frac{2}{1-\cos\theta}},$$

$$\cot\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{1+\cos\theta}{\sin\theta} = \frac{\sin\theta}{1-\cos\theta}.$$

Differentiation

$$\begin{aligned} D_x[u+v] &= u' + v' \\ D_x[u-v] &= u' - v' \\ D_x[uv] &= u'v + uv' \\ D_x\left[\frac{u}{v}\right] &= \frac{u'v - uv'}{v^2} \\ D_x[u^v] &= u^v\left(v'\ln u + \frac{vu'}{u}\right) \end{aligned}$$

Taylor's Theorem

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots$$

Lebniz's Theorem

$$\frac{d^n}{dx^n}\left[u(x)v(x)\right] = \sum_{k=0}^{n}\binom{n}{k} u^{(n-k}(x)$$

Walli's Formula

$$\int_0^{\pi/2} \sin^m x \cos^n x\, dx = \frac{(m-1)!!\,(n-1)!!}{(m+n)!!}\,k$$

Cauchy-Schwarz Inequality

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \le \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right)$$

Gamma Function

$$\Gamma(s) = \int_0^\infty t^{s-1}e^{-t}\,dt \quad (\Re(s) > 0)$$

Beta Function

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

"Mathematics is the music of reason."

— Dedicated to the beauty of numbers

Special Formulae

$$\tan\left(\frac{A+B}{2}\right) = \frac{\sin(A)+\sin(B)}{\cos(A)+\cos(B)} \qquad \text{(Napier's analogies)}$$

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2} \qquad \text{(Napier's rule)}$$

$$\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2} \qquad \text{(Gaussian integral)}$$

$$\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2} \qquad \text{(Dirichlet integral)}$$

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6} \qquad \text{(Basel problem)}$$

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx \qquad \text{(King property)}$$

$$T_n(x) = \cos(n \arccos x) \qquad \text{(Chebyshev Polynomial)}$$

Integration by Parts (ILATE Rule)

$$\int uv\,dx = u\int v dx - \int\left[D_x[u]\,.\int v dx\right]dx$$

ILATE Rule for choosing u:

- I Inverse trigonometric functions ($\sin^{-1}x, \tan^{-1}x, \ldots$)
- L Logarithmic functions ($\ln x$)
- A Algebraic functions (x^n)
- T Trigonometric functions ($\sin x, \cos x, \ldots$)
- E Exponential functions (e^x, a^x)

Choose u in this priority order and the other part as dv.

Finally, from

The Man Who Knew Infinity

Srinivasa Ramanujan

$$\cos\left(\frac{2\pi}{5}\right) = \frac{\sqrt{5}-1}{4}, \qquad \sin\left(\frac{2\pi}{5}\right) = \sqrt{\frac{5+\sqrt{5}}{8}}$$

$$\frac{\cos(x)}{\cos(2x)} = \prod_{n=1}^{\infty}\left(1 - \frac{x^2}{\left(\left(n-\frac{1}{2}\right)\pi\right)^2}\right)$$

$$\int_0^\infty x^{s-1} f(x)\,dx = \Gamma(s)\,\phi(-s)$$

$$\int_0^\infty \frac{\sin(ax)}{e^{2\pi x}-1}\,dx = \frac{1}{4}\coth\left(\frac{a}{2}\right) - \frac{1}{2a}, \quad (a > 0)$$

$$\tan^{-1}(x) = \sum_{k=0}^{\infty}\frac{2^{2k}(k!)^2}{(2k+1)!}\cdot\frac{x^{2k+1}}{(1+x^2)^{k+1}}$$

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801}\sum_{n=0}^{\infty}\frac{(4n)!(1103+26390n)}{(n!)^4\,396^{4n}}$$