25MT103: Linear Algebra

Module 1: Advanced Problems

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Module 1 - Advanced Problems

Advanced Questions

- Show that performing one elementary operation can change the determinant's value by a predictable scalar factor. Demonstrate with an example.
- Construct a system $A\mathbf{x} = \mathbf{b}$ where $\text{rank}(A) = 2$, $\text{rank}([A|\mathbf{b}]) = 3$.
- Create two linear systems sharing the same A but different $\mathbf{b}$, one consistent, one inconsistent.
- Solve a 4×4 system where one equation is the sum of others — discuss consistency.
- Given $A \in \mathbb{R}^{3\times 3}$ with $\det(A) = 0$, design a consistent system with infinite solutions.
- Design a parameterized system $A(k)\mathbf{x} = \mathbf{b}$ whose consistency depends on k.
- Given $A = LU$, solve three different RHS vectors efficiently using LU.
- Investigate geometric meaning of rank deficiency in $\mathbb{R}^3$.
- Derive A^{-1} using Gauss–Jordan and verify $AA^{-1} = I$.

Advanced Questions

- Implement a symbolic elimination for a general 3×3 system (derive formulas for x, y, z in terms of a_{ij}, b_i).
- Given a 4×4 matrix with symbolic entries, determine conditions under which it has exactly two pivot positions.
- Construct a 3×3 matrix whose rank changes depending on a parameter k. Analyze the transition values of k.
- Use Gauss–Jordan method to compute A^{-1} for a given 3×3 symbolic matrix. Analyze computational cost.
- Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 1 \end{pmatrix}$, determine rank analytically and geometrically.
- Compute A^{-1} using LU decomposition rather than Gauss–Jordan and compare numerical effort.

Theoretical Exploration

- Prove or disprove: For any nonsingular A, performing Gauss–Jordan on $[A|I]$ and extracting $I|A^{-1}$ is equivalent to solving $AX = I$ via LU factorization.
- Compare computational efficiency among Gaussian elimination, Gauss-Jordan method and LU decomposition methods for $n = 50$.

Thank You!

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