25MT103: Linear Algebra

Module 2: Advanced Problems

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Module 2 - Advanced Problems

Advanced Questions

- Find all subspaces of $\mathbb{R}^2$.
- Show that rank(A) = rank(A^T) for any real matrix.
- If A and B are similar, i.e. $B = P^{-1}AP$, prove that $\text{rank}(A) = \text{rank}(B)$.
- Show that for any $A \in \mathbb{R}^{m\times n}$, A^TA and AA^T have the same nonzero eigenvalues.
- Show that the projection matrix onto $\text{Col}(A)$ is $P = A(A^TA)^{-1}A^T$ when A has full column rank.
- Show that if $A = U\Sigma V^T$, then $\|A\|_2 = \sigma_{\max}(A)$, where $\sigma_{\max}$ is the largest singular value.

Theoretical Exploration

- Show that the intersection of two subspaces is a subspace.
- Prove that any orthogonal set of nonzero vectors is linearly independent.
- Show that any two bases of a finite-dimensional vector space have the same number of elements.
- Prove the Cauchy–Schwarz inequality for vectors in $\mathbb{R}^n$.
- Explain geometrically how SVD represents a linear transformation.
- Prove that if A is invertible and $A = QR$, then $A^{-1} = R^{-1}Q^T$.
- Prove that the rank of A equals the number of nonzero singular values of A.
- For a matrix A, explain how the SVD generalizes diagonalization when A is not symmetric.
- Show that the determinant of a linear transformation matrix equals the product of its eigenvalues.
- Let A be symmetric. Prove that there exists an orthogonal matrix Q such that $A = Q\Lambda Q^T$.

Thank You!

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