25MT103

LINEAR ALGEBRA

UNIT -1: MATRICES

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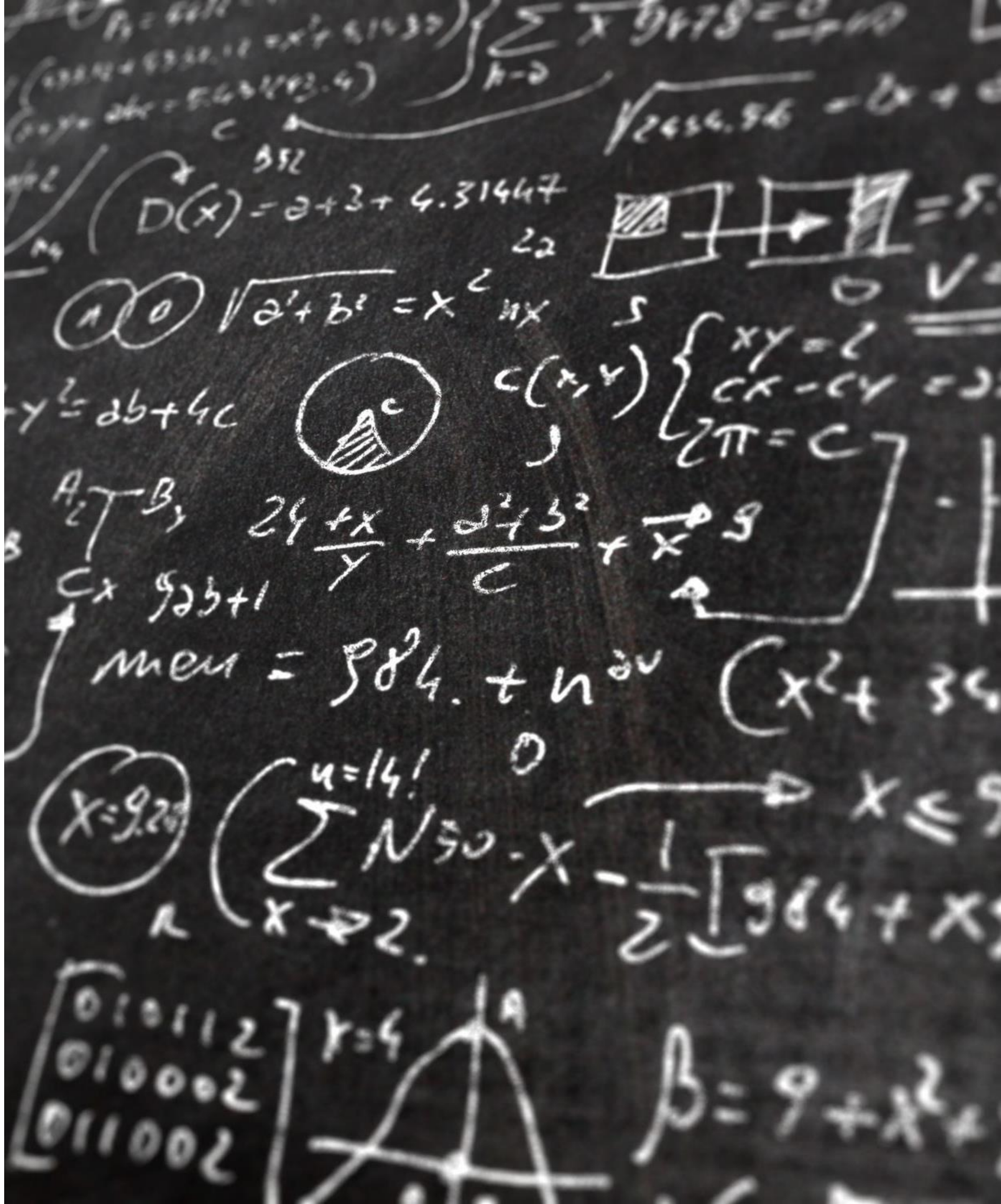
(Deemed to be University) - Estd. u/s 3 of UGC Act 1956



MATRICES:

Elementary row and column operations, Elementary matrices, Similar Matrices, Echelon form, Row reduced echelon form, Rank of a matrix, Inverse of a matrix by Gauss-Jordan method, LU decomposition.

SYLLABUS

EXAMPLE 3.7 Solve the following system:

$$\begin{aligned} x + 2y - 3z &= 1 \\ 2x + 5y - 8z &= 4 \\ 3x + 8y - 13z &= 7 \end{aligned}$$

We solve the system by Gaussian elimination.

Part A. (Forward Elimination) We use the coefficient 1 of x in the first equation L_1 as the pivot in order to eliminate x from the second equation L_2 and from the third equation L_3. This is accomplished as follows:

(1) Multiply L_1 by the multiplier $m = -2$ and add it to L_2; that is, "Replace L_2 by $-2L_1 + L_2$."

(2) Multiply L_1 by the multiplier $m = -3$ and add it to L_3; that is, "Replace L_3 by $-3L_1 + L_3$."

The two steps yield

$$\begin{aligned} x + 2y - 3z &= 1 \\ y - 2z &= 2 \\ 2y - 4z &= 4 \end{aligned} \qquad \text{or} \qquad \begin{aligned} x + 2y - 3z &= 1 \\ y - 2z &= 2 \end{aligned}$$

(The third equation is deleted, because it is a multiple of the second equation.) The system is now in echelon form with free variable z.

Part B. (Backward Elimination) To obtain the general solution, let the free variable $z = a$, and solve for x and y by back-substitution. Substitute $z = a$ in the second equation to obtain $y = 2 + 2a$. Then substitute $z = a$ and $y = 2 + 2a$ into the first equation to obtain

$$x + 2(2 + 2a) - 3a = 1 \qquad \text{or} \qquad x + 4 + 4a - 3a = 1 \qquad \text{or} \qquad x = -3 - a$$

Thus, the following is the general solution where a is a parameter:

$$x = -3 - a, \quad y = 2 + 2a, \quad z = a \qquad \text{or} \qquad u = (-3 - a,\ 2 + 2a,\ a)$$

EXAMPLE 3.22 Suppose $A = \begin{bmatrix} 1 & 2 & -3 \\ -3 & -4 & 13 \\ 2 & 1 & -5 \end{bmatrix}$. We note that A may be reduced to triangular form by the operations

"Replace R_2 by $3R_1 + R_2$"; "Replace R_3 by $-2R_1 + R_3$"; and then "Replace R_3 by $\frac{3}{2}R_2 + R_3$"

That is,

$$A \sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & 4 \\ 0 & -3 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & 4 \\ 0 & 0 & 7 \end{bmatrix}$$

This gives us the classical factorization $A = LU$, where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & -\frac{3}{2} & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & 4 \\ 0 & 0 & 7 \end{bmatrix}$$

Example. For what values of $a \in \mathbb{R}$, the following system of equations has (i) no solution (ii) a unique solution or infinitely many solutions

$$\begin{aligned} x + 2y + z &= 2 \\ 2x - 2y + 3z &= 1 \\ x + 2y - az &= a. \end{aligned}$$

Sol. No solution for $a = -1$ and unique solution for $a \neq -1$.