25MT103: Linear Algebra

Unit 2: Systems of Linear Equations

Dr. D Bhanu Prakash
Course Page: dbhanuprakash233.github.io/LA

Assistant Professor,
Department of Mathematics and Statistics.
Contact: db_maths@vignan.ac.in.
dbhanuprakash233.github.io.

VIGNAN'S FOUNDATION FOR SCIENCE, TECHNOLOGY & RESEARCH
(Deemed to be University) - Estd. u/s 3 of UGC Act 1956

Linear Systems - Tutorial

Syllabus

- Systems of Linear Equations
- Matrix Representation
- Consistency using rank
- Gaussian Elimination
- Gauss-Jordan method
- Do-little method

Consistency of system

Discuss the consistency of solutions of the system:

- $3x+4y+5z=a,\ 4x+5y+6z=b,\ 5x+6y+7z=c.$
- $(\lambda-1)x+(3\lambda+1)y+2\lambda z=0,\ (\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0,\ 2x+(3\lambda+1)y+3(\lambda-1)z=0.$ (Hint: Homogeneous system has infinite solutions if the determinent of coefficient matrix is zero).
- $x+y+z=2,\ 2x+2y+2z=4,\ x+y+z=3.$
- $x-2y+3t=2,\ 2x+y+z+t=-4,\ 4x-3y+z+7t=8.$ (Note: Change -2y in first equation to +2y and the consitency changes from inconsistent system to infinite number of solutions. Even a single sign change matters.)

Solve Linear Systems

1. Solve by Gaussian elimination:
   $x + 2y + z = 5,\ 3x + y - 2z = 4,\ 2x + 3y + 4z = 10.$
2. Determine consistency and solve if possible:
   $x + y + z = 2,\ 2x + 2y + 2z = 4,\ x + y + z = 3.$
3. Use Gauss–Jordan to find RREF and solution for:
   $2x + 4y - 2z = 2,\ -x - y + z = -1,\ x + 2y + 3z = 7.$
4. Compute LU (Doolittle) for $A = \begin{pmatrix} 2 & -1 & 1 \\ 4 & 1 & -1 \\ -2 & 2 & 3 \end{pmatrix}$ and solve

   $A\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$

Reasoning

Suppose B is a 3×3 matrix of rank 2. A new matrix C is obtained by multiplying the second column by 5 and adding it to the third column. Will the rank of C necessarily remain 2? Give a logical explanation and verify your claim using an example matrix.

Solution: Rank remains 2.

Reason: Replacing a column by itself plus a scalar multiple of another column is an elementary column operation (a column addition); such column operations preserve the rank.

Thank You!

Dr. D Bhanu Prakash
dbhanuprakash233.github.io
Mail: db_maths@vignan.ac.in