25MT103: Linear Algebra

Unit 4: Real Vector Space

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Real Vector Space - Tutorial

Linear Dependence and Span

1. Determine if the vectors $(1,2,3)$, $(2,4,6)$, and $(0,1,1)$ are linearly independent.
2. Find span$\{(1,0,0),(0,1,0)\}$ in $\mathbb{R}^3$.
3. Express $(3,3)$ as a linear combination of $(1,1)$ and $(1,2)$.
4. Determine if $(2,4,3)$ belongs to the span of $\{(1,0,1),(0,1,1)\}$.

Vector Spaces and Subspaces

1. Determine if the following sets are subspaces. If so, calculate the basis and dimension.
    1. $V = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + 3z = 0\}$
    2. $W = \{f : \mathbb{R} \to \mathbb{R} \mid f(0) = 0\}$
    3. $W = \{f : \mathbb{R} \to \mathbb{R} \mid f(0) = 1\}$
    4. $S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2\}$
    5. $W = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$
    6. $V = \{p(x) \in P_3 : p(1) = 0\}$
2. Determine if the set of all symmetric 2×2 matrices forms a subspace of $\mathbb{R}^{2\times 2}$.

Bases and Dimension

1. Find a basis for the null space of $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$.
2. Find a basis of P_2 (polynomials of degree ≤ 2) and compute coordinates of $p(x) = 1 + 2x + x^2$ relative to that basis.
3. Find bases for the row space and column space of
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 1 \end{bmatrix}.$$
4. Given
$$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},$$
verify whether A and B are similar and compare their ranks and eigenvalues.
5. Find the dimension of the space of 2×2 symmetric matrices.

Null Space, Orthogonal Projections, and Rank–Nullity

1. Compute the null space of
$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \end{bmatrix}.$$
Also verify the Rank–Nullity Theorem.
2. Find the orthogonal projection of $b = (2,3,4)^T$ onto the subspace spanned by
$$v_1 = (1,0,0)^T, \quad v_2 = (1,1,0)^T.$$
3. Prove that $\mathrm{Col}(A)$ and $\mathrm{Nul}(A^T)$ are orthogonal complements in $\mathbb{R}^m$.
4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x,y) = (x+2y, 3x+y)$. Find its matrix representation and determine if T is invertible.

Hints and Key Steps

For deeper practice:

- Always verify closure under vector operations.
- When finding spans, solve $a_1v_1 + \cdots + a_kv_k = w$ for w.

Thank You!

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