25MT103: Linear Algebra

Unit 5: Real Inner Product Space

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Real Inner Product Space - Tutorial

Inner Products and Norms

1. Compute $\|v\|$ for $v = (3, 4, 12)$ using the standard inner product.
2. Check if $(1, -1, 0)$ and $(1, 1, 0)$ are orthogonal.
3. In $\mathbb{R}^3$ with the inner product $\langle x, y \rangle = x^T A y$, where

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

   compute $\|x\|_A$ for $x = (1, 2, 3)^T$.
4. Verify that $\langle f, g \rangle = \int_0^1 f(x) g(x)\, dx$ defines an inner product on $C[0, 1]$.

Orthogonalization and Decomposition

1. Apply the Gram–Schmidt process to $(1,1,0)$, $(1,0,1)$, $(0,1,1)$.
2. Find the QR decomposition of the following matrices.
    1. $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$.
    2. $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.
    3. $A = \begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix}$
3. Compute the singular value decomposition (SVD) of the following matrices.
    1. $A = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$.
    2. $A = \begin{bmatrix} 3 & 0 \\ 4 & 0 \end{bmatrix}$.

Orthogonalization and Decomposition

1. Use QR decomposition to solve the least-squares problem $Ax = b$ where

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}.$$

2. Find a rank-1 approximation to

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

   using its SVD.

Hints and Key Steps

For deeper practice:

- For Gram–Schmidt, compute projections carefully.
- In QR, note that $R = Q^T A$.
- For SVD, compute $A^T A$ and its eigenvalues.

Thank You!

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