

Practice Problems on Vector Space and Subspace

Section 7, 14, 21

Faculty: Dr. D Bhanu Prakash

Problems on Vector Space

A Vector Space = (Set + 2 Operations) should satisfy 10 conditions.

Problem 1: Determine whether the set of all 2×2 matrices with real entries forms a vector space under standard matrix addition and scalar multiplication.

Problem 2: Let V be the set of all polynomials of degree exactly 3 (i.e., $p(x) = ax^3 + bx^2 + cx + d$ where $a \neq 0$). Verify whether V forms a vector space under standard polynomial addition and scalar multiplication.

Problem 3: Consider the set $W = \{(x, y, z) \in \mathbb{R}^3 : x + 2y - z = 0\}$. Verify whether W is a vector space under standard vector addition and scalar multiplication in \mathbb{R}^3 .

Problem 4: Let $V = \{(x, y) \in \mathbb{R}^2 : xy \geq 0\}$ (the set of all points in the first and third quadrants, including axes). Determine if V forms a vector space under standard addition and scalar multiplication.

Answers

Problem 1: Yes

Problem 2: No

Problem 3: Yes

Problem 4: No

Problems on Subspace

A subset of a vector space is a subspace if it contains the additive identity and satisfy the closure of vector addition and scalar multiplication.

Problem 1: Verify whether the set $S = \{(x, y, z) \in \mathbb{R}^3 : 2x - y + 3z = 0\}$ is a subspace of \mathbb{R}^3 .

Problem 2: Determine if $W = \{(a, b, c) \in \mathbb{R}^3 : a = 2b \text{ and } c = -b\}$ is a subspace of \mathbb{R}^3 .

Problem 3: Let H be the set of all polynomials in P_3 (polynomials of degree at most 3) such that $p(1) = 0$. Verify whether H is a subspace of P_3 .

Problem 4: Consider $U = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 1\}$. Determine whether U is a subspace of \mathbb{R}^3 .

Problem 5: Let V be the set of all 2×2 diagonal matrices. Verify whether V is a subspace of $M_{2 \times 2}$ (the vector space of all 2×2 matrices).

Problem 6: Determine if $W = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ (the closed unit disk) is a subspace of \mathbb{R}^2 .

Answers

Problem 1: Yes

Problem 2: Yes

Problem 3: Yes

Problem 4: No

Problem 5: Yes

Problem 6: No