SRI SATHYA SAI INSTITUTE OF HIGHER LEARNING
(Deemed to be University)
Department of Mathematics and Computer Science

TUTORIAL 1

Mathematical Review & Unconstrained Optimization

UMAT-503: Optimization Techniques	Instructor: Dr. D Bhanu Prakash	Date: 20-07-2026

Q1. (a) Using the definition of a convex set, prove that $S1 = \{x \in R^2 : x1 + 2x2 \leq 4, x1 \geq 0, x2 \geq 0\}$ is convex.

(b) Show, with a counterexample, that $S2 = \{x \in R^2 : x1^2 + x2^2 \geq 1\}$ is NOT convex.

(c) Define an ε-neighbourhood of a point x0 in R^n. Determine whether S1 is a polyhedron, and if so, express it as an intersection of half-spaces (hyperplanes).

[BTL-3]

Q2. Find the equation of the hyperplane in R^3 passing through the points P1 = (1, 0, 1), P2 = (0, 1, 1), and P3 = (1, 1, 0). Express the hyperplane in the linear-variety form $\{x : a^Tx = b\}$.

[BTL-3]

Q3. For $f(x, y) = x^2 + 2y^2$, find the gradient at the point (1, 1) and the equation of the level curve through (1, 1). Analyze and show that the gradient vector is perpendicular to the tangent of the level curve at that point.

[BTL-4]

Q4. Expand $f(x, y) = e^x \ln(1 + y)$ in a Taylor series about the point (0, 0), retaining terms up to second order.

[BTL-3]

Q5. For $f(x, y) = x^3 + y^3 - 3xy$, find all critical points. Using the first-order necessary condition and the second-order (Hessian) sufficient condition, classify each critical point as a local minimizer, local maximizer, or saddle point.

[BTL-4]

Q6. (a) Using the golden section search method, perform two iterations to reduce the interval of uncertainty for minimizing $f(x) = x^2 + 2x$ over the initial interval $[-3, 2]$.

(b) Briefly explain how the Fibonacci search method's interval-reduction sequence differs from that of golden section search, and state one advantage of Fibonacci search over golden section search.

[BTL-3]

Q7. (a) Apply the bisection method to locate the minimizer of $f(x) = e^x - 2x$ by finding the root of $f'(x) = 0$ on $[0, 1]$, performing three iterations.

(b) Apply Newton's method to the same function starting at $x0 = 0$, performing two iterations.

(c) Analyze and compare the convergence behaviour (rate, robustness, and assumptions) of the two methods.*[BTL-4]*

— End of Tutorial —