SRI SATHYA SAI INSTITUTE OF HIGHER LEARNING

(Deemed to be University)

Department of Mathematics and Computer Science

CIE - I - SOLUTIONS

Topics: Mathematical Review & Unconstrained Optimization

UMAT-503: Optimization Techniques	Instructor: Dr. D Bhanu Prakash	Date: 08-08-2026

Q1. Newton's Method

Textbook 1 - Page 117 - Till x(2). x(1) = 0.7552, x(2) = 0.7391

Q2. SONC - Textbook 1 - Page 90

Q3. Derivative (Jacobian) matrix

$F(x1,x2) = (x1^2x2,\ x1+x2^2,\ \sin(x1x2))$

Jacobian matrix:

$J(x1,x2) = [\ [2x1x2, x1^2], [1, 2x2], [x2\cos(x1x2), x1\cos(x1x2)]\ ]$

Evaluated at (1, 0):

Row 1: $[2(1)(0), 1^2] = [0, 1]$ Row 2: $[1, 2(0)] = [1, 0]$ Row 3: $[0\cdot\cos(0), 1\cdot\cos(0)] = [0, 1]$

$J(1,0) = [\ [0, 1], [1, 0], [0, 1]\ ]$

Q4. Conditions for local minimizers

$f(x,y) = x^3 + y^3 - 3xy$

First-order necessary condition ($\nabla f = 0$):

$fx = 3x^2 - 3y = 0 \Rightarrow y = x^2$ $fy = 3y^2 - 3x = 0 \Rightarrow x = y^2$

Substituting $y = x^2$ into $x = y^2$: $x = x^4 \Rightarrow x^4 - x = 0 \Rightarrow x(x^3 - 1) = 0$
So x = 0 or x = 1. $x=0 \Rightarrow y=0$. $x=1 \Rightarrow y=1$.
Critical points: (0,0) and (1,1).

Second-order (Hessian) sufficient condition:

$H(x,y) = [\ [fxx, fxy], [fyx, fyy]\ ] = [\ [6x, -3], [-3, 6y]\ ]$

At (0,0): $H = [\ [0,-3], [-3,0]\ ]$, $\det(H) = (0)(0) - (-3)(-3) = -9 < 0$

$\Rightarrow$ Indefinite Hessian $\Rightarrow$ (0,0) is a SADDLE POINT.

At (1,1): $H = [\ [6,-3], [-3,6]\ ]$, $\det(H) = 36 - 9 = 27 > 0$, and $f_{xx} = 6 > 0$
$\Rightarrow$ Positive definite Hessian $\Rightarrow$ (1,1) is a LOCAL MINIMIZER.