SRI SATHYA SAI INSTITUTE OF HIGHER LEARNING
(Deemed to be University)
Department of Mathematics and Computer Science

CIE-I SOLUTIONS

Topics: Elements of Probability & Discrete Random Variables

UDSC-201: Probability and Statistics	Instructor: Dr. D Bhanu Prakash	Date: 07-08-2026

Q1. Total Probability and Bayes Theorem

For any events A and B, $P(B)= P(B \cap A)+ P(B \cap A')= P(B \mid A)P(A)+ P(B \mid A')P(A')$

Bayes' theorem → rule for **updating probability** based on new evidence

Prior probability: belief before evidence; **Posterior probability**: belief after evidence

Bayes' theorem = **bridge** connecting prior → posterior using likelihood of evidence

Q2. Independence and the addition rule

Given: $P(A) = 0.4$, $P(B) = 0.5$, $P(A\cap B) = 0.2$

Check independence: A and B are independent iff $P(A\cap B) = P(A)\cdot P(B)$.

$P(A)\cdot P(B) = 0.4 \times 0.5 = 0.20 = P(A\cap B) \Rightarrow$ A and B ARE independent.

Addition rule: $P(A \cup B) = P(A) + P(B) - P(A\cap B) = 0.4 + 0.5 - 0.2 = 0.7$

If A and B were mutually exclusive instead: then $P(A\cap B) = 0$, so $P(A \cup B) = P(A) + P(B) - 0 = 0.4 + 0.5 = 0.9$

Analysis: mutually exclusive events with nonzero probabilities can never be independent, because independence requires $P(A\cap B) = P(A)P(B) = 0.2 \neq 0$. So the two conditions (given $P(A\cap B)=0.2$, and mutually exclusive) are mutually inconsistent — this illustrates that 'independent' and 'mutually exclusive' are fundamentally different (in fact opposite) relationships between events.

Q3. Mean - Binomial Random Variable - Textbook - Page 53-54

Q4. PMF, CDF, mean and variance

Given: $P(X=x) = k(x+1)$, for $x = 0,1,2,3,4$

Find k: Sum of probabilities = 1 $\quad$ $k(1+2+3+4+5) = 15k = 1 \Rightarrow k = 1/15$

PMF table: $P(0)=1/15$, $P(1)=2/15$, $P(2)=3/15$, $P(3)=4/15$, $P(4)=5/15$

CDF F(x):

$F(0) = 1/15 \approx 0.067$

$F(1) = 3/15 = 0.20$
$F(2) = 6/15 = 0.40$
$F(3) = 10/15 \approx 0.667$
$F(4) = 15/15 = 1$

Mean: $E(X) = \Sigma\, x \cdot P(x) = (0 \cdot 1 + 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5)/15 = 40/15 = 8/3 \approx 2.667$
$E(X^2)$: $= (0 \cdot 1 + 1 \cdot 2 + 4 \cdot 3 + 9 \cdot 4 + 16 \cdot 5)/15 = 130/15 = 26/3$
Variance: $\mathrm{Var}(X) = E(X^2) - [E(X)]^2 = 26/3 - (8/3)^2 = 26/3 - 64/9 = (78-64)/9 = 14/9 \approx 1.556$