SRI SATHYA SAI INSTITUTE OF HIGHER LEARNING

(Deemed to be University)

Department of Mathematics and Computer Science

TUTORIAL-II

Topics: Continuous Probability Distributions

UDSC-201: Probability and Statistics	Instructor: Dr. D Bhanu Prakash	Date: 19-08-2026

Question 1

[BTL-3: Apply — compute k, CDF, probability, mean and variance from a given pdf]

A continuous random variable X has probability density function

$$f(x) = k\,x,\ \ 0 \le x \le 4, \quad \text{and } f(x) = 0 \text{ otherwise.}$$

(a) Find the value of k. (b) Find the CDF F(x). (c) Find $P(1 \le X \le 3)$. (d) Find the mean and variance of X.

Question 2

[BTL-3: Apply — Uniform distribution probability and moments]

A bus arrives at a stop at a uniformly random time between 9:00 AM and 9:20 AM. Let X (in minutes after 9:00) denote the arrival time, so $X \sim \text{Uniform}(0, 20)$. Find (a) the pdf of X, (b) $P(5 < X < 15)$, (c) the mean and variance, and (d) the probability that a passenger who arrives at 9:08 waits more than 5 minutes.

Question 3

[BTL-3: Apply — Exponential distribution and the memoryless property]

The lifetime of an electronic component (in years) follows an Exponential distribution with mean life 5 years. (a) Find the rate parameter λ and the pdf. (b) Find the probability the component survives more than 8 years. (c) Given that it has already survived 4 years, find the probability it survives 4 more years, and comment on what property this illustrates.

Question 4

[BTL-3: Apply — Normal distribution, standardization (Z-scores)]

Scores on a standardized test are normally distributed with mean $\mu = 500$ and standard deviation $\sigma = 100$. (a) Find the probability that a randomly chosen student scores between 450 and 650. (b) Find the minimum score needed to be in the top 10% of students.

Question 5

[BTL-4: Analyze — justify and apply the Normal approximation to the Binomial with continuity correction]

A fair coin is tossed $n = 400$ times. Let X be the number of heads. (a) Justify whether a Normal approximation is appropriate. (b) Using the Normal approximation with continuity correction, find $P(X \geq 210)$. (c) Compare briefly what would go wrong if continuity correction were omitted.

Question 6

[BTL-3: Apply — Erlang distribution as a sum of Exponential stages]

Customers arrive at a service counter according to a Poisson process at rate $\lambda = 4$ per hour. Let T be the time until the 3rd customer arrives, so $T \sim \text{Erlang}(k = 3, \lambda = 4)$. (a) Write the pdf of T. (b) Find E(T) and Var(T). (c) Find the probability that the 3rd customer arrives within 30 minutes (0.5 hour).

Question 7

[BTL-4: Analyze — recognize Gamma model parameters and relate to component distributions]

The total repair time (in hours) for a machine is modeled as $X \sim \text{Gamma}(\text{shape } \alpha = 2.5, \text{rate } \beta = 0.5)$. (a) Write the pdf of X. (b) Find the mean and variance. (c) Explain how this Gamma model relates to the Exponential and Erlang distributions, and state the condition under which it reduces to each.

Question 8

[BTL-4: Analyze — interpret Weibull shape parameter and compute reliability/failure rate behavior]

The lifetime (in years) of a mechanical part follows a Weibull distribution with shape parameter $\beta = 2$ and scale parameter $\eta = 4$. (a) Write the pdf and reliability function. (b) Find the probability the part survives beyond 5 years. (c) Determine whether the failure rate is increasing, constant, or decreasing with age, and justify using the shape parameter.

Question 9

[BTL-4: Analyze — apply the Lognormal model via the underlying Normal transform]

The size of insurance claims X (in thousand \$) is modeled as Lognormal, meaning $\ln(X) \sim N(\mu = 1.5, \sigma^2 = 0.36)$. (a) Find the mean and variance of X itself (not ln X). (b) Find the probability a claim exceeds \$10,000. (c) Explain why the Normal model is applied to ln(X) rather than to X directly.

Question 10

[BTL-4: Analyze — apply the Beta distribution to a bounded proportion and interpret its shape]

The proportion X of defective items in a large shipment (a value always between 0 and 1) is modeled with a Beta distribution with parameters $\alpha = 2$, $\beta = 5$. (a) Write the pdf of X. (b) Find the mean and variance. (c) Determine the mode of the distribution and interpret what it says about typical defect proportions. (d) Explain why Beta (rather than Normal) is the natural choice here.