SRI SATHYA SAI INSTITUTE OF HIGHER LEARNING

(Deemed to be University)

Department of Mathematics and Computer Science

TUTORIAL-I SOLUTIONS

Topics: Elements of Probability & Discrete Random Variables

UDSC-201: Probability and Statistics	Instructor: Dr. D Bhanu Prakash	Date: 17-08-2026

Q1. Committee selection (counting & probability)

Given: 7 men, 6 women, total 13 people. Choose a committee of 5 with at least 3 women.

Case-wise counting (addition rule for disjoint cases):

Exactly 3 women, 2 men: $C(6,3) \times C(7,2) = 20 \times 21 = 420$
Exactly 4 women, 1 man: $C(6,4) \times C(7,1) = 15 \times 7 = 105$
Exactly 5 women, 0 men: $C(6,5) \times C(7,0) = 6 \times 1 = 6$

Total ways (at least 3 women) = $420 + 105 + 6 = 531$

Probability of exactly 3 women:

Total ways to choose any 5 from 13 = $C(13,5) = 1287$
P(exactly 3 women) = $420 / 1287 = 140/429 \approx 0.3263$

Q2. Bayes' formula (machines and defects)

Given: $P(M1)=0.30$, $P(M2)=0.45$, $P(M3)=0.25$
$P(D|M1)=0.02$, $P(D|M2)=0.03$, $P(D|M3)=0.01$

Total probability of a defect:

$$\begin{aligned} P(D) &= P(M1)P(D|M1) + P(M2)P(D|M2) + P(M3)P(D|M3) \\ &= (0.30)(0.02) + (0.45)(0.03) + (0.25)(0.01) \\ &= 0.006 + 0.0135 + 0.0025 = 0.022 \end{aligned}$$

Bayes' formula:

$P(M2|D) = P(M2)P(D|M2) / P(D) = 0.0135 / 0.022 \approx 0.6136$

So there is about a 61.4% chance a defective item came from machine M2.

Q3. Hypergeometric distribution

Given: $N = 20$ items, $K = 5$ defectives, sample $n = 4$ drawn without replacement.

Model choice: Because sampling is without replacement from a finite population split into two categories (defective/non-defective), the number of defectives in the sample follows the Hypergeometric distribution (not Binomial, since trials are not independent — probabilities change as items are removed).

PMF: $P(X=x) = C(K,x) \cdot C(N-K, n-x) / C(N,n)$

For x = 2:
$P(X=2) = C(5,2) \cdot C(15,2) / C(20,4)$
$C(5,2)=10$, $C(15,2)=105$, $C(20,4)=4845$
$P(X=2) = (10 \times 105)/4845 = 1050/4845 \approx 0.2167$

Q4. Negative Binomial distribution

Given: $p = 0.7$ (success probability per free throw), want the probability that the 4th success ($r = 4$) occurs exactly on the 6th attempt ($k = 6$).

PMF of Negative Binomial: $P(X=k) = C(k-1, r-1) \cdot p^r \cdot (1-p)^{(k-r)}$

$P(X=6) = C(5,3) \times (0.7)^4 \times (0.3)^2$
$C(5,3) = 10$, $0.7^4 = 0.2401$, $0.3^2 = 0.09$
$P(X=6) = 10 \times 0.2401 \times 0.09 = 0.21609$

Mean and variance of number of attempts:
$E(X) = r/p = 4/0.7 \approx 5.714$
$Var(X) = r(1-p)/p^2 = 4(0.3)/0.49 = 1.2/0.49 \approx 2.449$

Q5. Geometric distribution and the memoryless property

Given: probability of failure on any day $p = 0.1$, geometric distribution for day of first failure.

(a) P(first failure on day 5):
$P(X=5) = (1-p)^4 \cdot p = (0.9)^4 (0.1) = 0.6561 \times 0.1 = 0.06561$

(b) Conditional probability given survival for 3 days:
By the memoryless property of the Geometric distribution:
$P(X > 3+2 \mid X > 3) = P(X > 2)$
$P(X > 2) = (1-p)^2 = (0.9)^2 = 0.81$

Analysis: The probability of surviving 2 more days, given the machine has already survived 3 days without failure, equals 0.81 — exactly the same as the unconditional probability of surviving 2 days from day one. This confirms the Geometric distribution is memoryless: past survival gives no information about future failure risk, since each day is an independent Bernoulli trial with the same failure probability.

Q6. Design/Evaluate — two-stage inspection model (BTL-6)

Step 1 — Sample space and events:

Let the sample space Ω consist of outcomes for a single inspected item: whether it is defective or not, and whether each stage detects it. Define:

D = event the item is actually defective, $P(D) = p$ (known defect rate)

T1 = event Stage-1 detects the defect, with $P(T1|D) = p1$

T2 = event Stage-2 detects the defect (given it reaches Stage 2), with $P(T2|D) = p2$

Assume T1 and T2 are conditionally independent given D (the two inspection mechanisms act independently on the same item).

Step 2 — Probability of escaping detection:

An item escapes detection if it is defective AND neither stage catches it:

$P(\text{escape} \mid D) = (1 - p1)(1 - p2)$

By the multiplication rule (independence of stages) and total probability, the overall (unconditional) probability that a randomly inspected item is defective and escapes detection is:

$P(\text{escape} \cap D) = p\cdot(1 - p1)(1 - p2)$

Step 3 — Applying Bayes' theorem:

The probability an item is actually defective, given it passed both stages undetected, is:

$P(D \mid \text{escape}) = P(\text{escape}|D)\cdot P(D) / P(\text{escape})$

where $P(\text{escape}) = P(D)(1-p1)(1-p2) + P(D')\cdot 1$ [non-defective items always 'pass', assuming no false positives], i.e. $P(\text{escape}) = p(1-p1)(1-p2) + (1-p)$.

This quantifies the residual risk that a 'passed' item is still defective.

Step 4 — Choosing a distribution for items inspected until the first caught defect:

Let $q = p\cdot[1 - (1-p1)(1-p2)]$ is the probability that a given item is BOTH defective AND caught by at least one stage. If items are inspected independently and q is constant across items, then the number of items inspected until the first one is caught as defective follows a **Geometric distribution** with parameter q: $P(N=n) = (1-q)^{(n-1)}\cdot q$.

Binomial is rejected because it counts successes in a fixed number of trials, not the waiting time to the first success. Negative Binomial would apply if we wanted the trial number of the r-th caught defect ($r > 1$); since we want only the first, Geometric (the special case $r = 1$) is the appropriate and most parsimonious choice.

Step 5 — Critical evaluation of assumptions:

- Constant defect rate p across all items may not hold if defects cluster (e.g., a bad batch), violating the i.i.d. assumption needed for the Geometric model.
- Independence of the two inspection stages may fail in practice if both stages share a common cause of error (e.g., poor lighting affecting both visual inspections).

- No false positives were assumed for non-defective items; in reality inspection stages may have false-alarm rates, which would require extending the model with additional conditional terms.

- The model treats each item independently; if inspectors fatigue over time, detection probabilities p1, p2 may drift, breaking the constant-parameter assumption underlying both the Bayes' calculation and the Geometric distribution.